Key for a Hidden Quantum State

или как без знания исходного состояния узнать какое состояние получится внутри резонатора, непрерывно измеряя его изменение энергии

V. P. Stefanov , V. N. Shatokhin ,D. S. Mogilevtsev , and S. Ya. Kilin,
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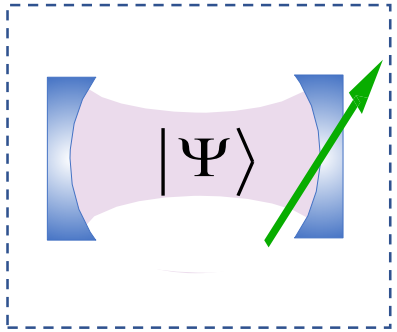
Plan

- Introduction
- Quantum compound trajectories and stochastic processes
- Scheme of proof-of-principle experiment
- Conditional state evolution (two theorems)
- Energy-to-time decoding: time scaling and energy-time uncertainty relation
- K4HQS Protocol
- The created state as a hidden resource
- Non-ideal detection and protocol feasibility
- Interpretation and discussion

Introduction

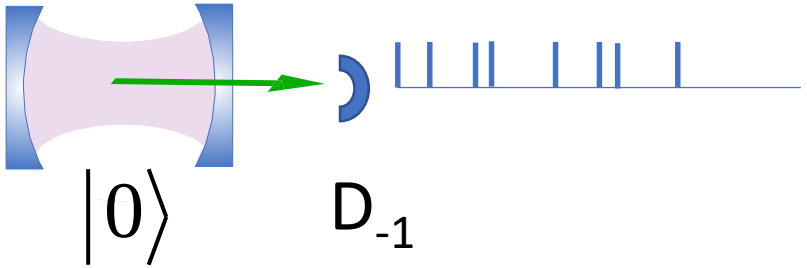


One copy of **unknown** state → No cloning theorem

How to use unknown state? → To transform to **a known nontrivial state**

(But not, how to know ?)

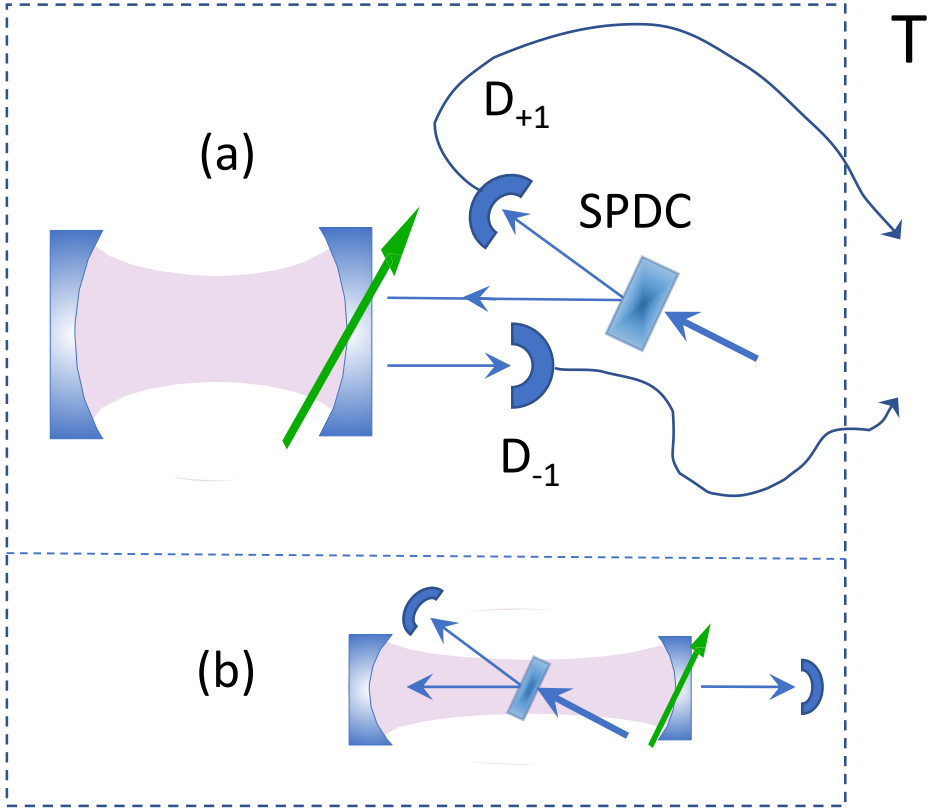
QND → $|n\rangle$



Vacuum initialization

Introduction

Continuous measurement of the cavity energy change



The measurement record

$K_N=\{\xi_j, t_j\}$

$$\zeta_j \in \pm 1$$

$$\Delta^{(N)} = \sum_{j=1,\cdots,N} \zeta_j$$

The cumulative difference between the input and output photons

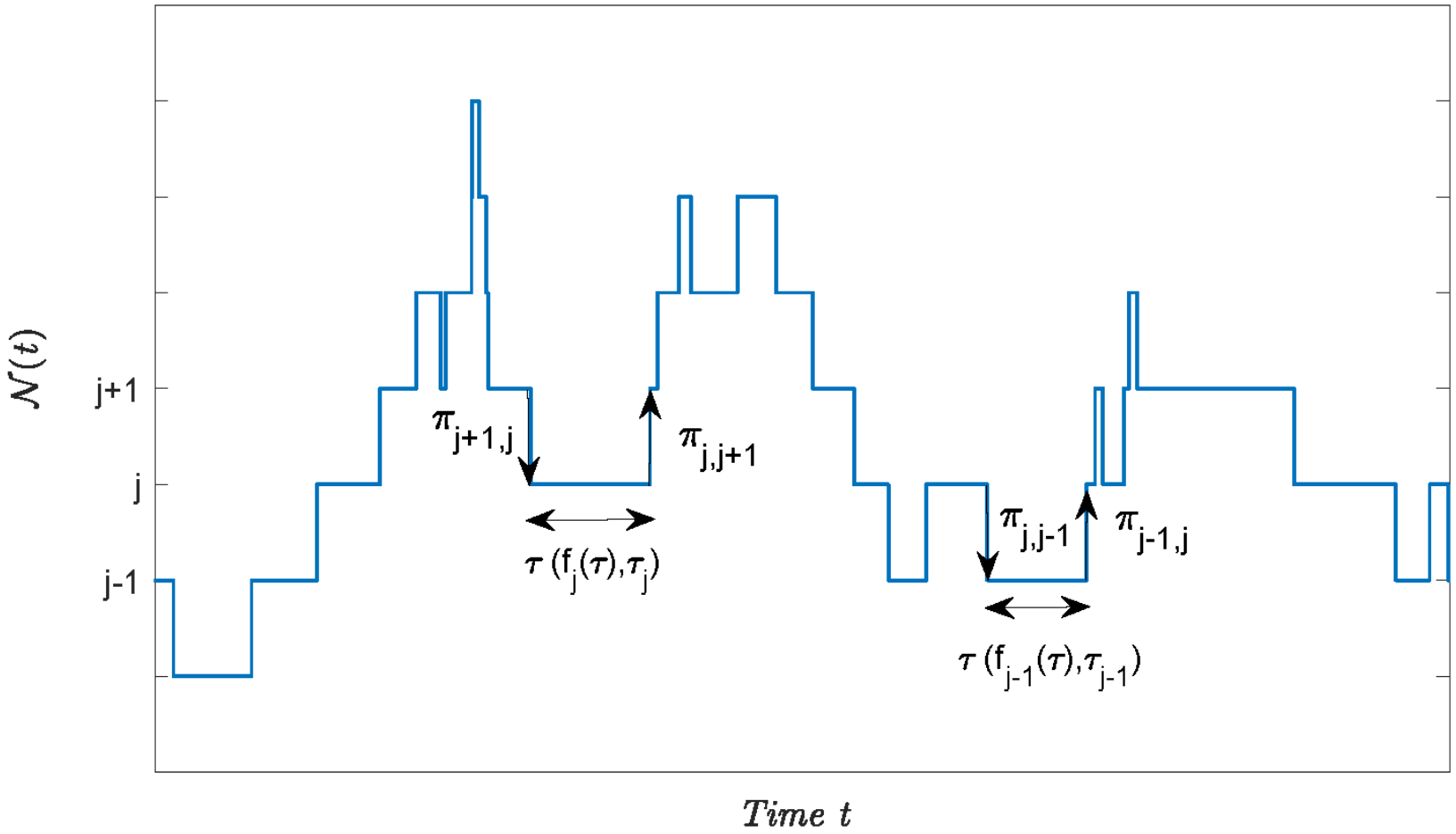
Vacuum initialization → CM $\quad |\Psi_N\rangle = |n\rangle, \quad n = \Delta^{(N)}$

→

~~Vacuum initialization~~ → CM $\quad |\Psi_N\rangle = ?$

→

Quantum compound trajectories and stochastic processes



A particular realization of the classical BD process

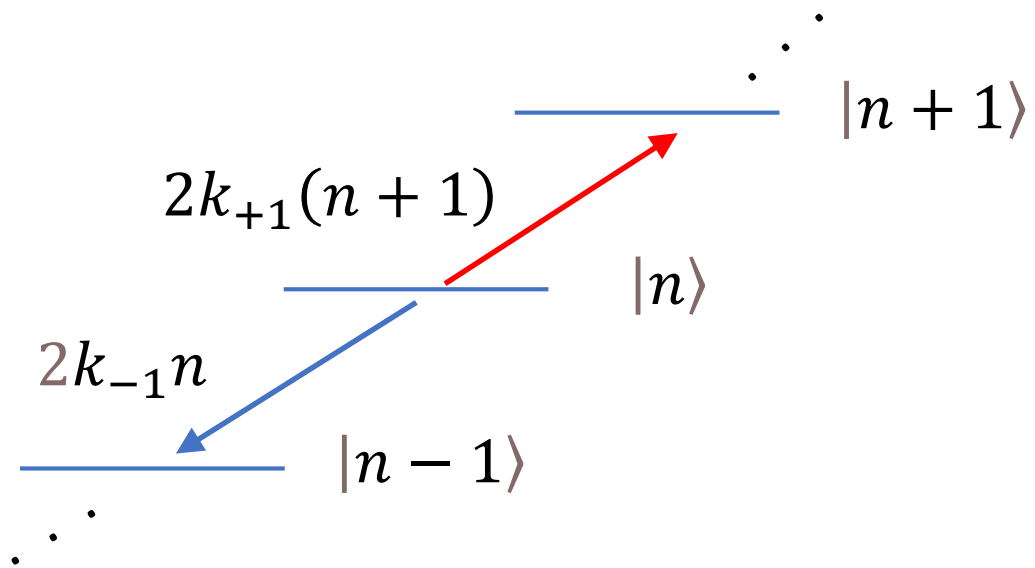
Quantum compound trajectories and stochastic processes

$$\dot{\rho} = \sum_{\xi=\pm 1} k_\xi ([a_\xi, \rho a_\xi^\dagger] + [a_\xi \rho, a_\xi^\dagger]).$$

$$\rho_{t\to\infty}^{(n)} \sim e^{-\frac{k_{+1}}{\Gamma} n}$$

Thermal state

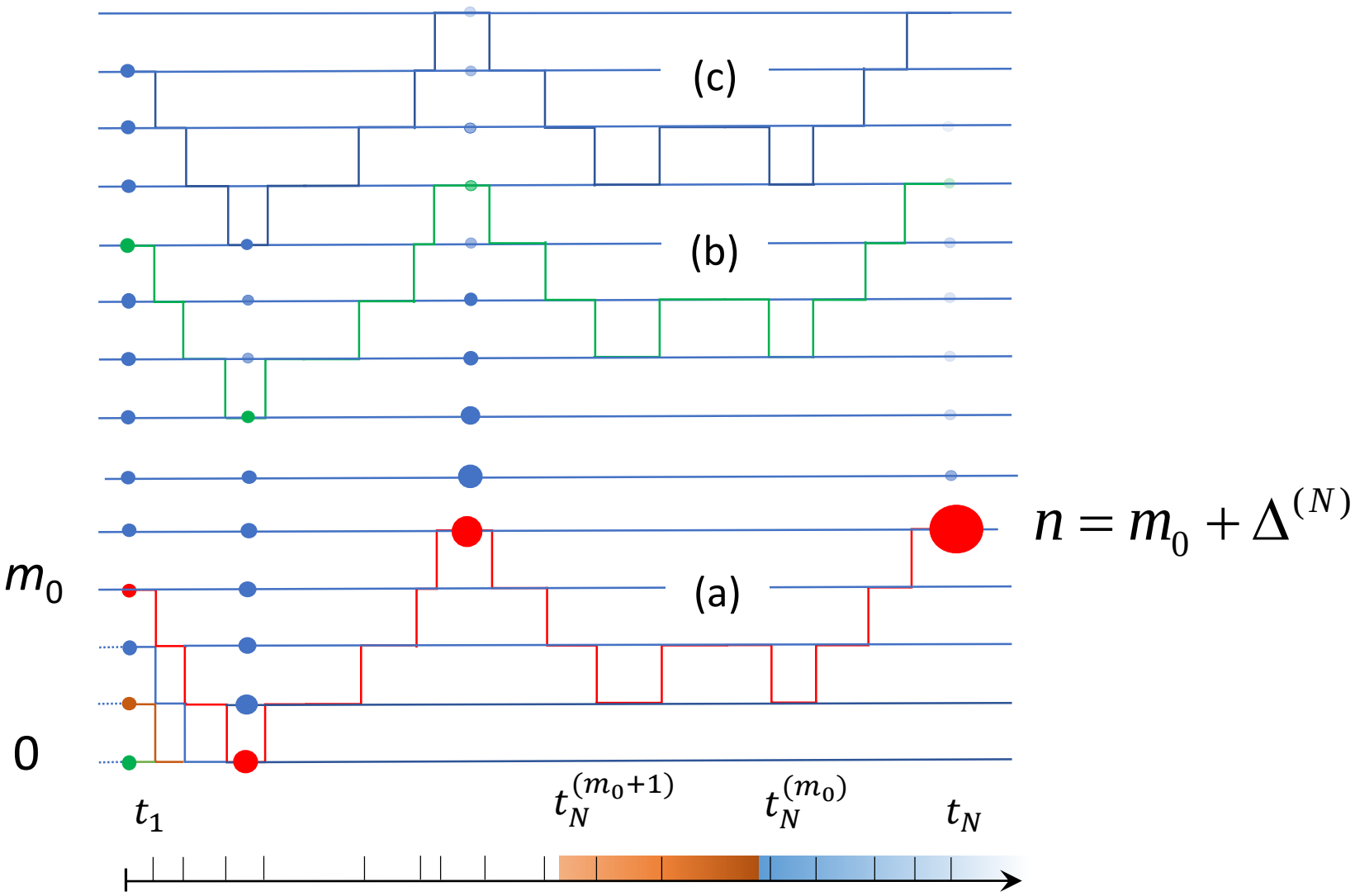
The state-dependent up/down transition rates



The waiting time distribution functions for the next jump

$$f_n(\tau) = e^{-\gamma_n \tau}, \qquad \gamma_n = 2(\Gamma n + k_{+1}), \quad \Gamma = k_{+1} + k_{-1}$$

Quantum compound trajectories and stochastic processes



Quantum compound trajectories and stochastic processes

$$\rho(\xi,t)=\frac{J_\xi S^{(t-t')}\rho(\xi',t')}{Tr[J_\xi S^{(t-t')}\rho(\xi',t')]}$$

The field state $\rho(\xi, t)$ at the moment t right after the ξ-count

ξ-count →

$$J_\xi x = 2k_\xi a_\xi x a_\xi^\dagger$$

No counts →

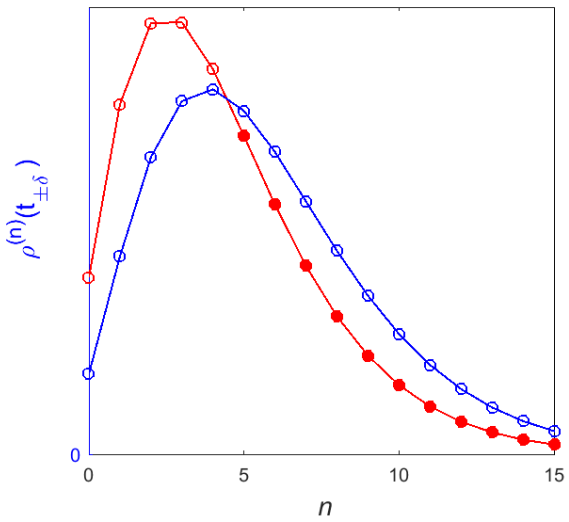
$$S^{(\tau)}x = B^{(\tau)}x\left(B^{(\tau)}\right)^\dagger$$

$$\begin{cases} B^{(\tau)} = e^{-iH_{eff}\tau/\hbar} \\ H_{eff} = -i\hbar\sum_{\xi=\pm1} k_\xi a_\xi^\dagger a_\xi \end{cases}$$

The same record presents subjectively different QTs

Conditional state evolution

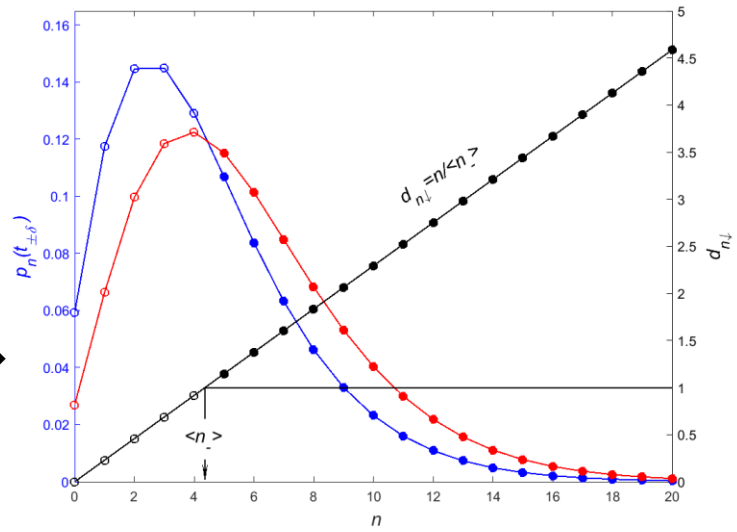
No counts
state evolution →



$$\frac{\dot{\rho}_S^{(n)}}{\rho_S^{(n)}} = -2\Gamma(n - \bar{n}_\tau)$$

$$\bar{n}_\tau = \sum_n n\rho_S^{(n)}(t' + \tau)$$

After count
state redistribution →



$$\frac{\rho^{(n+\xi)}(\xi,t)}{\rho_S^{(n)}(t)} = \frac{(2n+\xi+1)}{(2\bar{n}_{t-t'}+\xi+1)} = d_{n\xi}$$

Conditional state evolution (two theorems)

<u>Theorem 1</u>: Independently of the a priori state of an open cavity field mode, the field state after a long single run becomes, in ergodic case, a random Fock state.

<u>Theorem 2:</u> If during a long single run with a record $\{\xi_k, t_k\}$ of N counts a Fock state has been created, this state can be inferred with unit asymptotic fidelity for ergodic regime using only the sequence $\{\xi_k\}$ of counts' types and its total duration t_N without referring to the initial state and times $\{t_k\}$ of intermediate clicks.

Conditional state evolution (two theorems)

The DM for a sub-ensemble conditioned by the record $\{\xi k, tk\}$ of N counts.

$$\langle n | \rho^{(N)}_{\{\xi_k, t_k\}} | n \rangle = \delta_{n, m+\Delta^{(N)}} \mathfrak{A}^{(N)}_{m,\{\xi_k, t_k\}} \langle m | \rho_0 | m \rangle$$

m-trajectories
$\{\xi_k\} : |m\rangle \mapsto |n\rangle$

$$\Delta^{(N)} = \sum_{k=1}^{N} \xi_k$$

The measured change of the cavity energy in quanta

The conditional transition probability (CTP)

$$\mathfrak{A}^{(N)}_{m,\{\xi_k, t_k\}} = e^{-2\Gamma m t_N} (m)_{N,f} / \langle e^{-2\Gamma m t_N} (m)_{N,f} \rangle_{\rho_0}$$

no-jump maps

jump maps

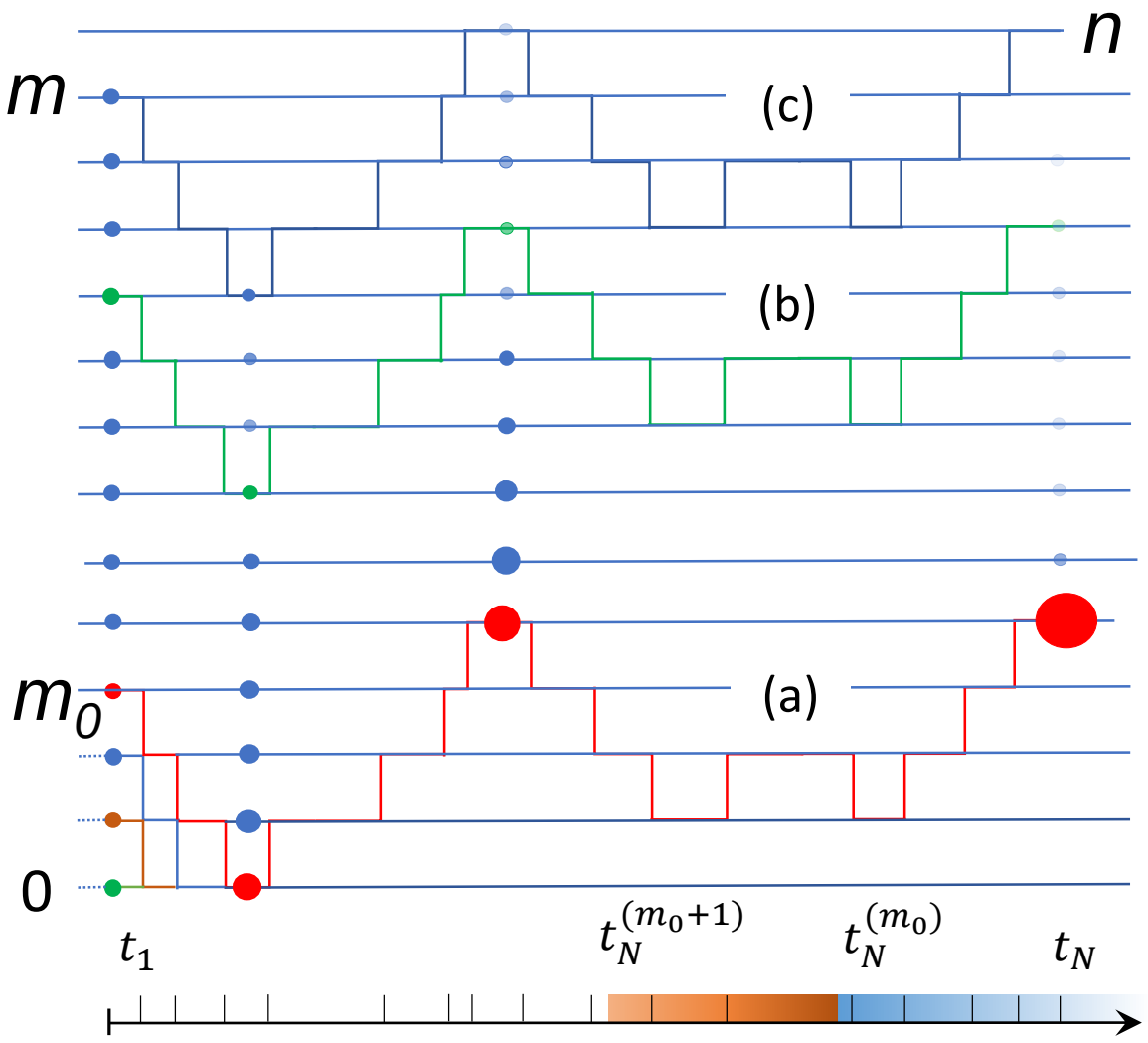
$$(m)_{N,f} = \prod_{k=1}^{N} \left(m + f^{(k)} \right)$$

$$f^{(k)} = \Delta^{(k)} + (1 - \xi_k)/2$$

f-key

Conditional state evolution (two theorems)

m-trajectories $\{\xi_k\} : |m\rangle \mapsto |n\rangle$



$$n = m + \Delta^{(N)}$$

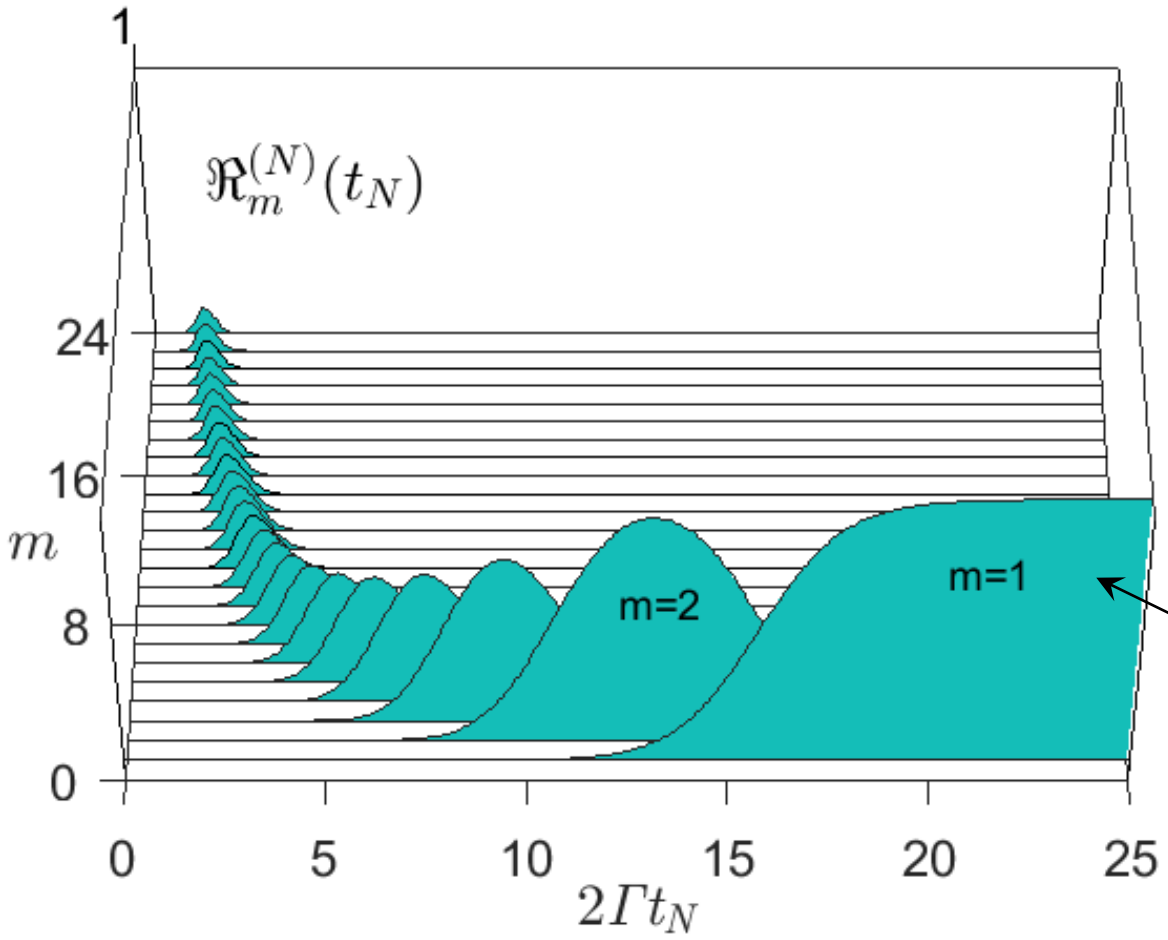
- *low weight trajectories*

- *dominant m_0-trajectory* $(m_0 = -f_{min} + 1)$

- *truncated trajectories*

$$t_N > t_N^{(m_0)} > T_0$$

Conditional state evolution (two theorems)



$$m_0 = -f_{min} + 1 \equiv -\Delta_{min}$$

$$t_N \gg t_N^{(m_0)}$$

$$2\Gamma t_N^{(m)} = \log\left(\frac{(m+1)_{N,f}}{(m)_{N,f}}\right)$$

$$\Re_{m_0}^{(N)}(t_N) = \frac{1}{1+e^{-2\Gamma(t_N - t_N^{(m_0)})}}$$

Logistic function

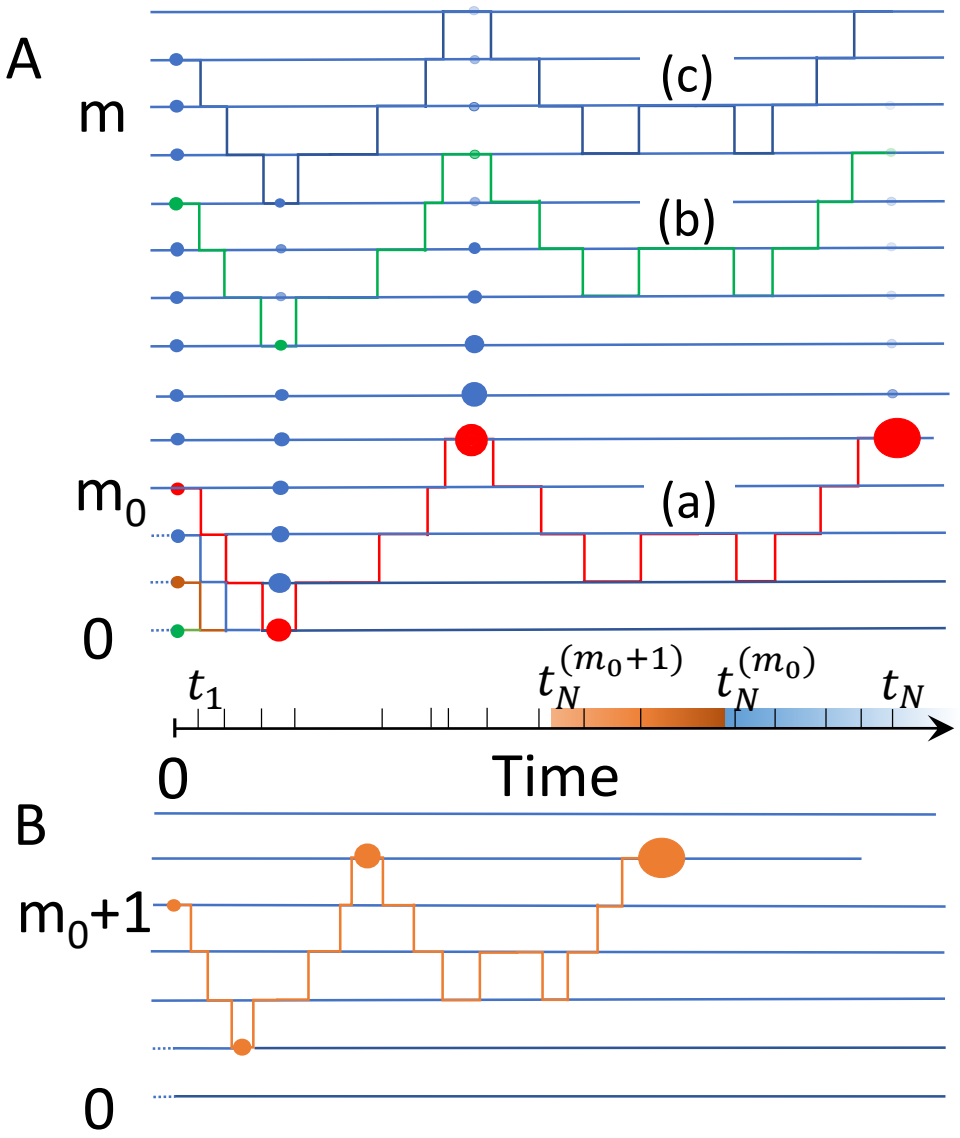
f_{40} =1100122112210000111234433443222111111100

Energy-to-time decoding:
time scaling and energy-time uncertainty relation

$n \rightleftarrows n + 1$

indistinguishable in frequency,

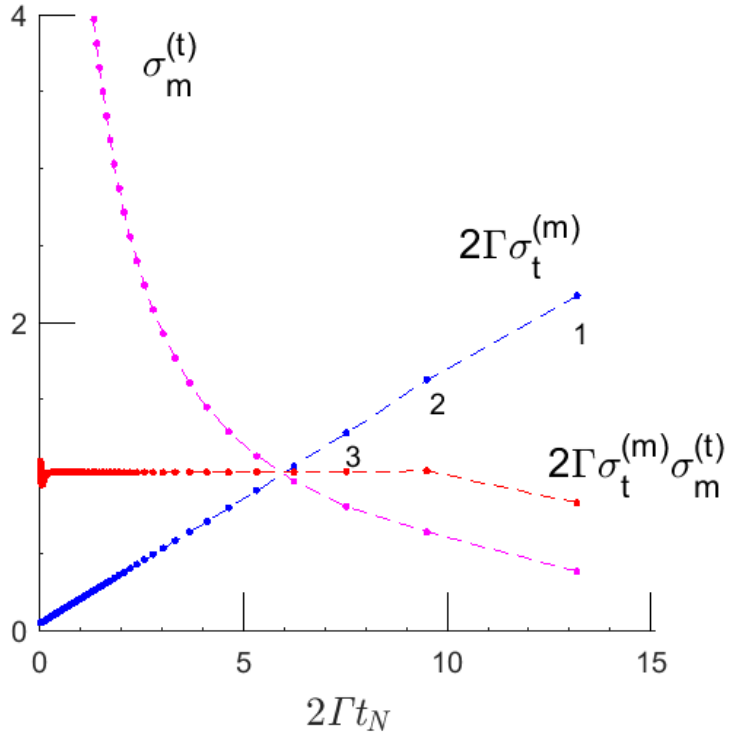
but can be discerned by transition rates $2k_{\xi}(n + 1)$



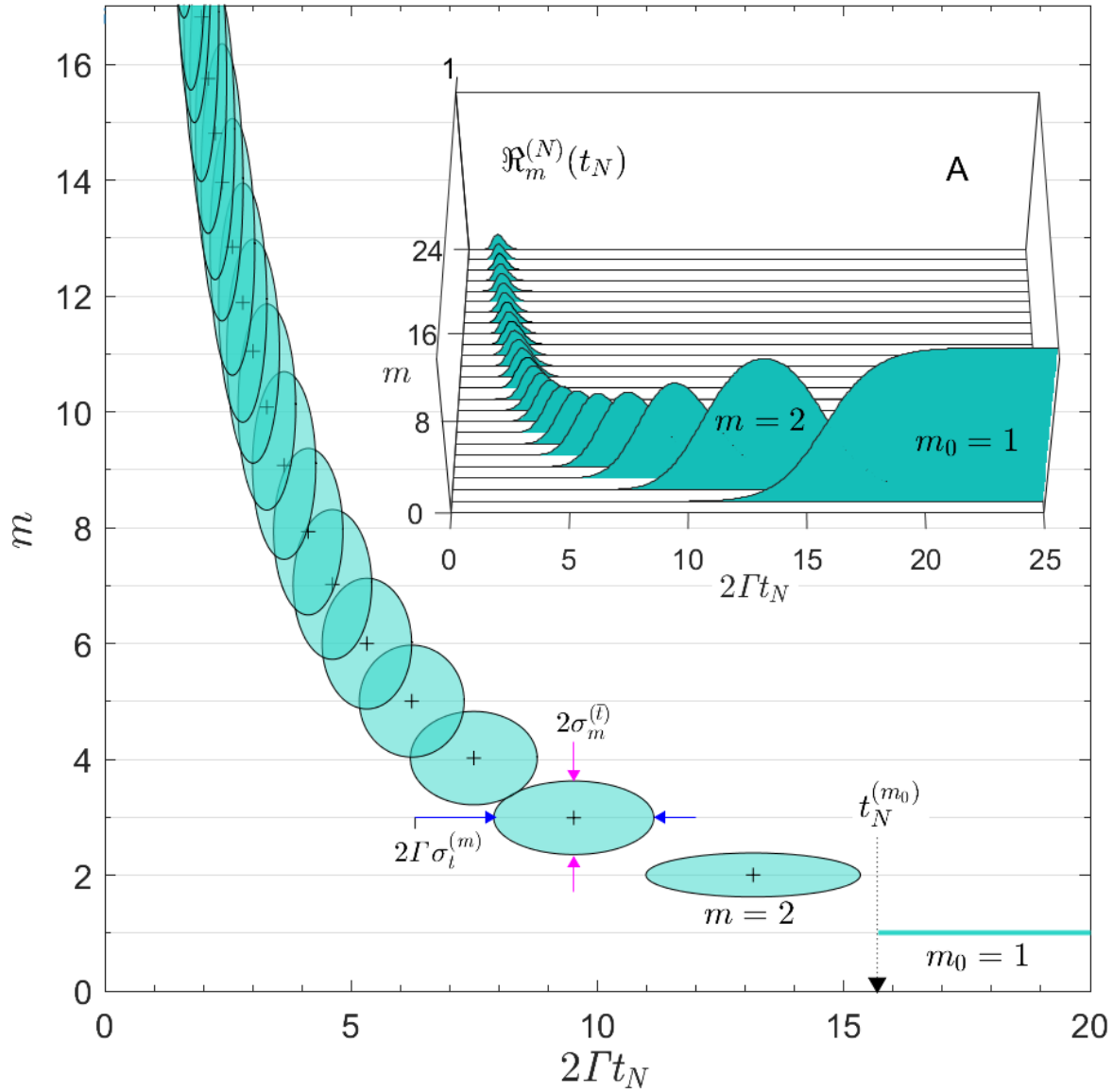
Time windows $m > m_0$

$$\mathbf{T}_N^{(m)} = (t_N^{(m+1)}, t_N^{(m)})$$

Energy-to-time decoding:
time scaling and energy-time uncertainty relation



$$\sigma_m^{(t)}\sigma_t^{(m)} = 1/2\Gamma$$



K4HQS Protocol

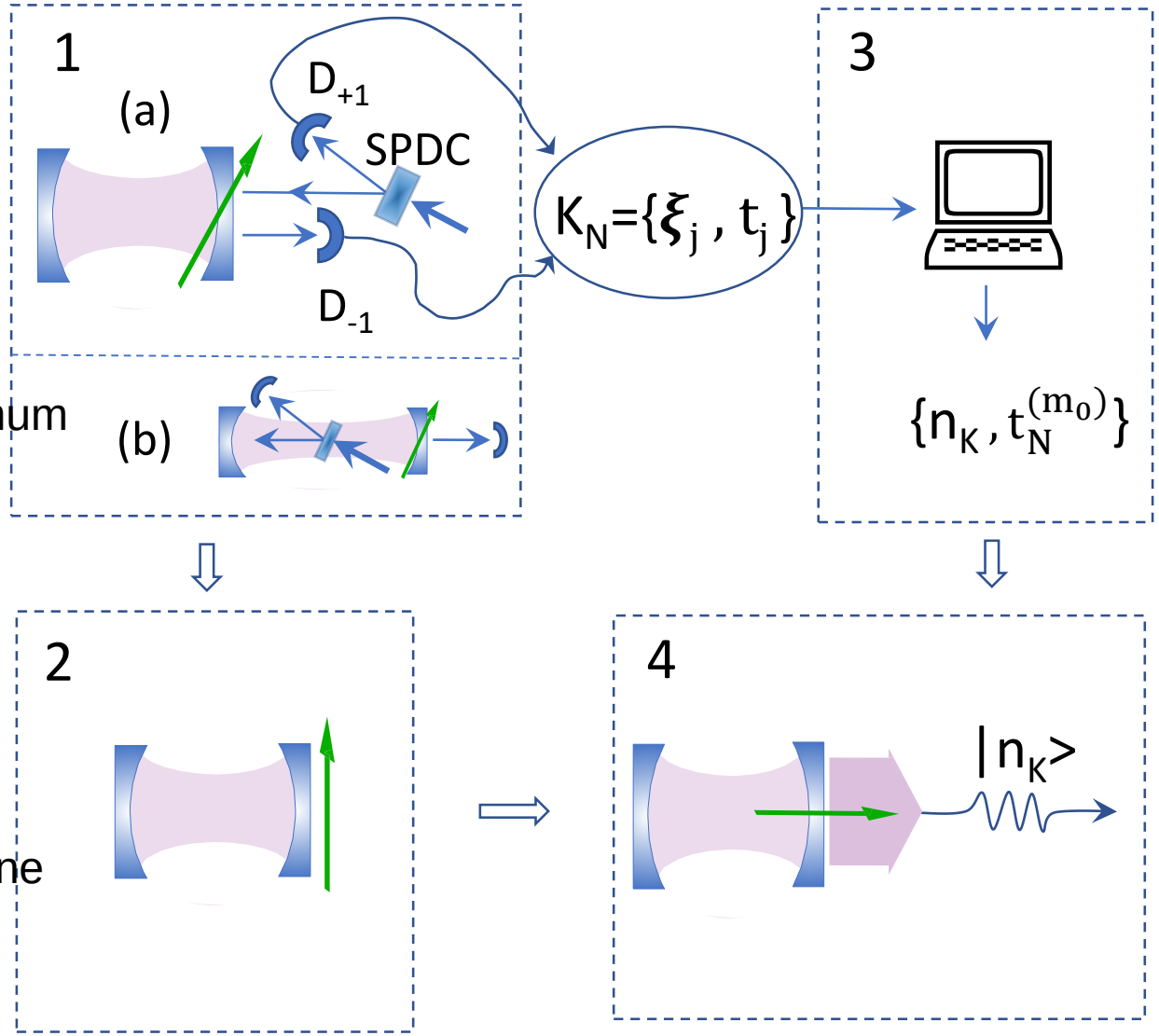
1. Register a random sequence $K_N = \{\xi_k, t_k\}$ of N clicks from ideal detectors monitoring photons leaving and entering the cavity.

2. ”Lock” the cavity from the environment to leave the in-cavity field unchanged.

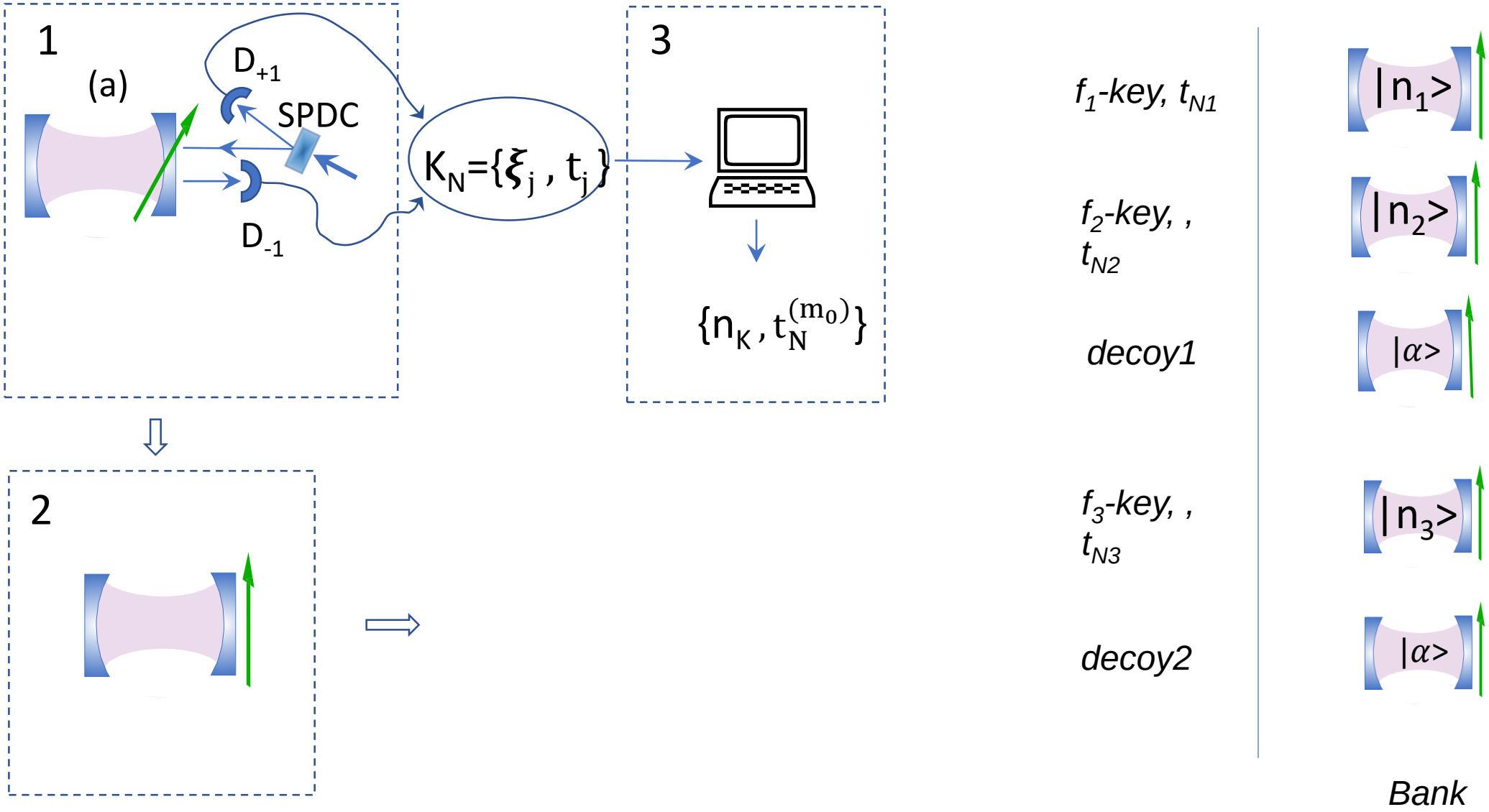
3. For the registered sequence $\{\xi_k\}$, calculate the minimum value f_{min} of the *f-key*, the cumulative index $\mathbf{A}^{(N)}$ and the threshold $t_N^{(m_0)}$

4. Provided that $t_N > t_N^{(m_0)}$ the intracavity field will be in the Fock state $|n_K = \mathbf{A}^{(N)} - f_{min} + 1\rangle$. Unlock the cavity to use this state .

5. If $t_N < t_N^{(m_0)}$, one can start the protocol over, or find the $\mathbf{T}_N^{(m)}$ -window wherein t_N is located, and determine m.



The created state as a hidden resource



Non-ideal detection and protocol feasibility

Protocol feasibility

$$T_{e0} = -(2k_{-1}(1-q))^{-1}\log(1-q)$$

First Passage Time from equilibrium to |0>

$$\langle t_N \rangle > t_N^{(m_0)} > T_{e0}$$

$$N < 50, \quad q = \frac{k_{+1}}{k_{-1}} < 0.75$$

Non-ideal detection

Detectors with error ϵ

$N\epsilon$ events will not be registered

Superconducting nanowire single-photon detectors with 99.5% detection efficiency *(J. Chang, et al, APL Photonics 6, 036114 (2021))*

$$F = 1 - N\epsilon \simeq 0.8 - 0.9$$

$$N = 40 - 20$$

Immunity to paired errors:

$Q_{0.05} = -0.88$, $Q_{0.1} = -0.76$

Interpretation and discussion

The considered model of **continuous measurement of the cavity energy change**, which in fact goes back to the origins of Planck's quantum theory, shows that as time goes by, the a posteriori quantum state has progressively larger overlap with a **random energy eigenstate**.

For **the ergodic regime** in the limit $t \to \infty$, the realized measurement becomes an orthogonal POVM measurement.

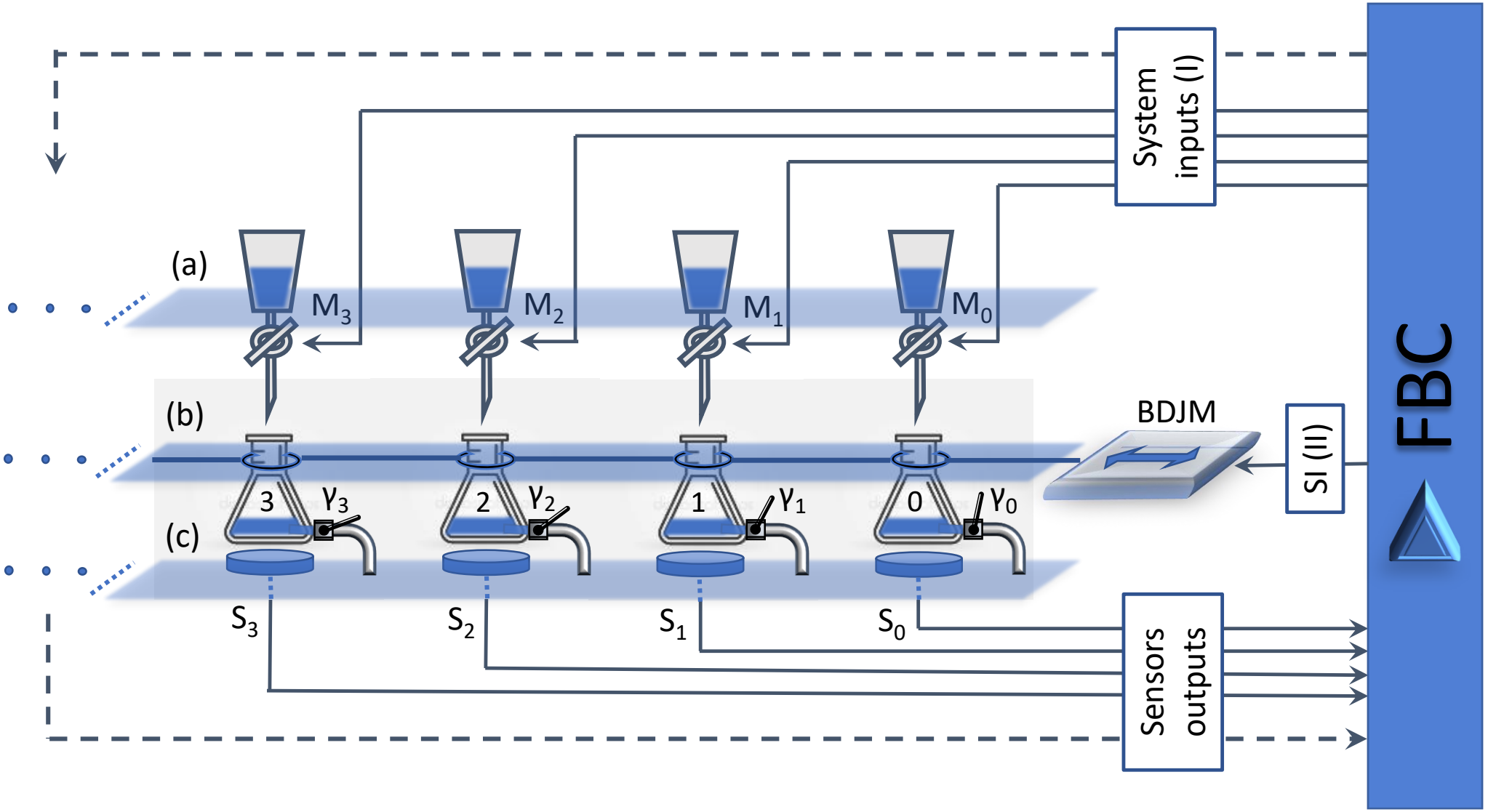
POVM measurement

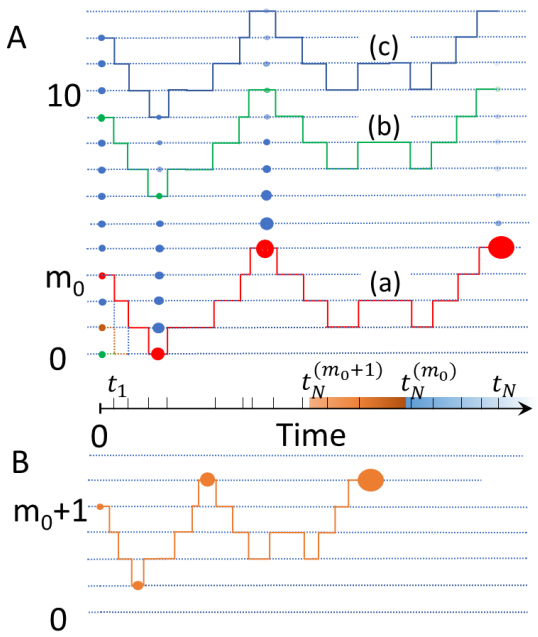
The two-integers outcome	$(\Delta^{(N)}, m_0 = -\Delta_{min})$
The measurement operators	$A^{(m_0)} = \lvert m_0 + \Delta^{(N)} \rangle \langle m_0 \rvert$
The orthogonal set of the POVM elements	$E^{(m_0)} = \lvert m_0 \rangle \langle m_0 \rvert, m_0 = 0,1,2,\ldots$ $\sum_{m_0} (A^{(m_0)})^\dagger A^{(m_0)} = \sum_{m_0} E^{(m_0)} = I$

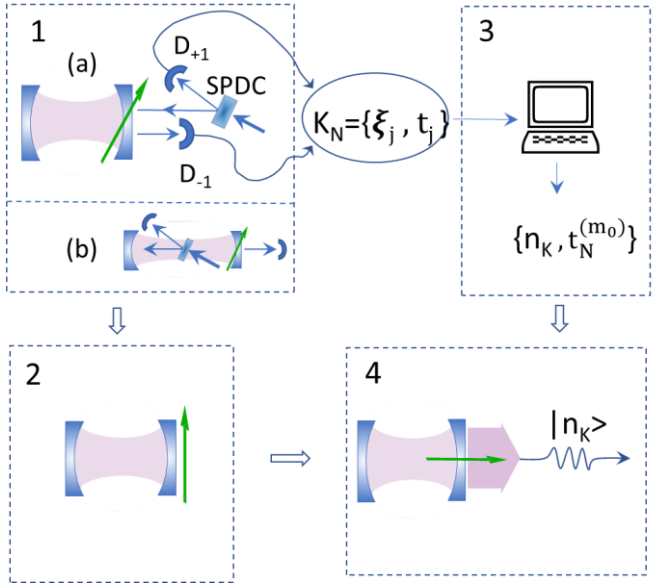
A posteriori state can be **asymptotically stable**, that is, independent of the a priori state. I.e. two initial states-of-knowledge (e.g., complete and limited) will converge together as data is obtained, iff both contain m_0

The considered quantum stochastic BD process has the classical counterpart, **the collective BD process**, which has not yet been discussed in the literature.

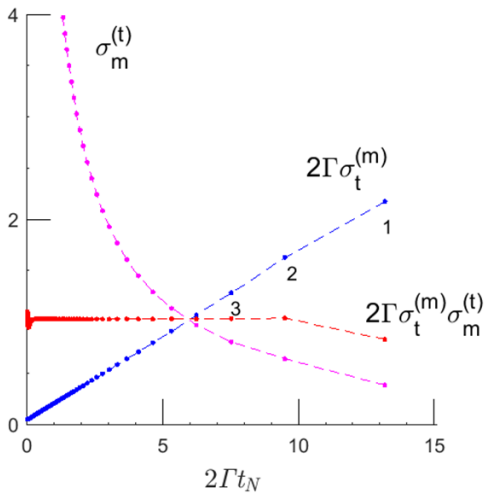
The collective BD process

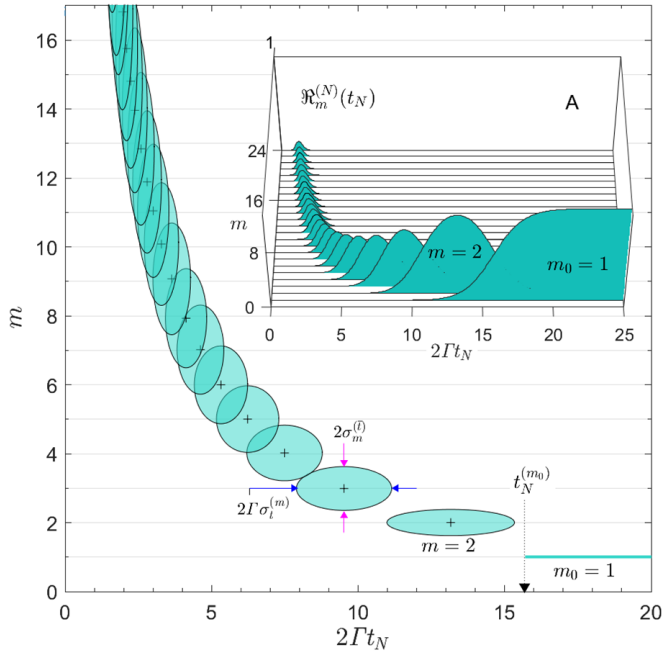



A
10
(c)
(b)
m_0
(a)
0
t_1
$t_N^{(m_0+1)}$
$t_N^{(m_0)}$
t_N
0
Time
B
m_0+1
0


1
(a)
D_{+1}
SPDC
D_{-1}
$K_N=\{\xi_j, t_j\}$
3
$\{n_K, t_N^{(m_0)}\}$
(b)
2
4
$|n_K>$

Thank you!


$\sigma_m^{(t)}$
$2\Gamma\sigma_t^{(m)}$
$2\Gamma\sigma_t^{(m)}\sigma_m^{(t)}$
1
2
3
$2\Gamma t_N$


m
$2\Gamma t_N$
$\Re_m^{(N)}(t_N)$
A
$m = 2$
$m_0 = 1$
$2\sigma_m^{(l)}$
$2\Gamma\sigma_l^{(m)}$
$t_N^{(m_0)}$
$m = 2$
$m_0 = 1$