Generating single-photon qudits via optical nonlinear effects

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Nonclassical light sources

Single-photons	Quantum communications
Two-photon entangled states	Quantum computing
NOON states	Quantum imaging
Cluster sates	Quantum metrology
Squeezed states	…
…	

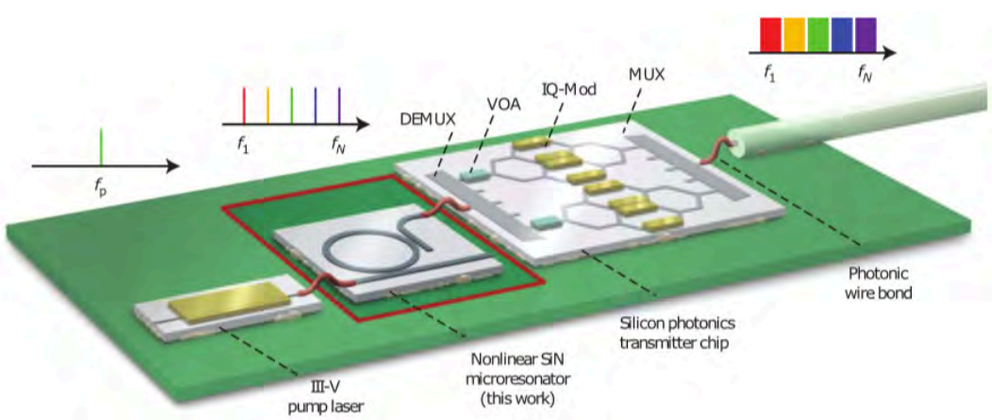


Image: M. Borghi, et al. J. Opt., 2017

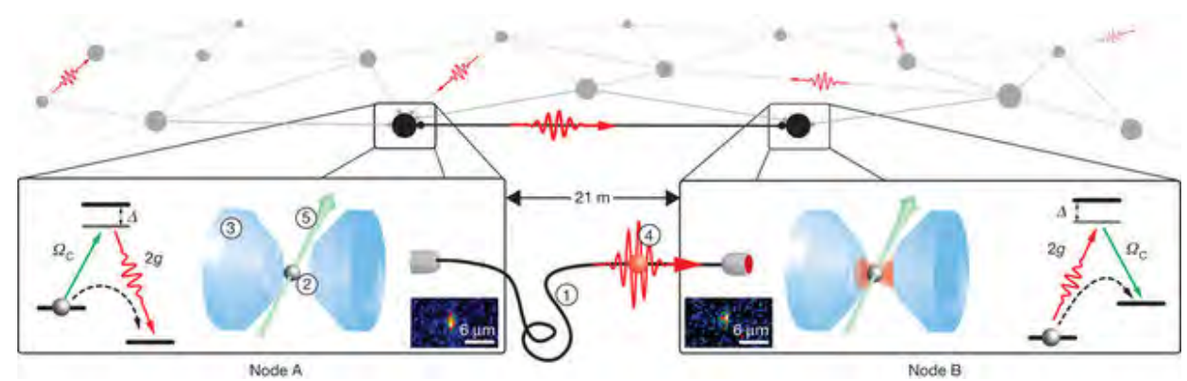


Image: S. Ritter at al., Nature, 2012

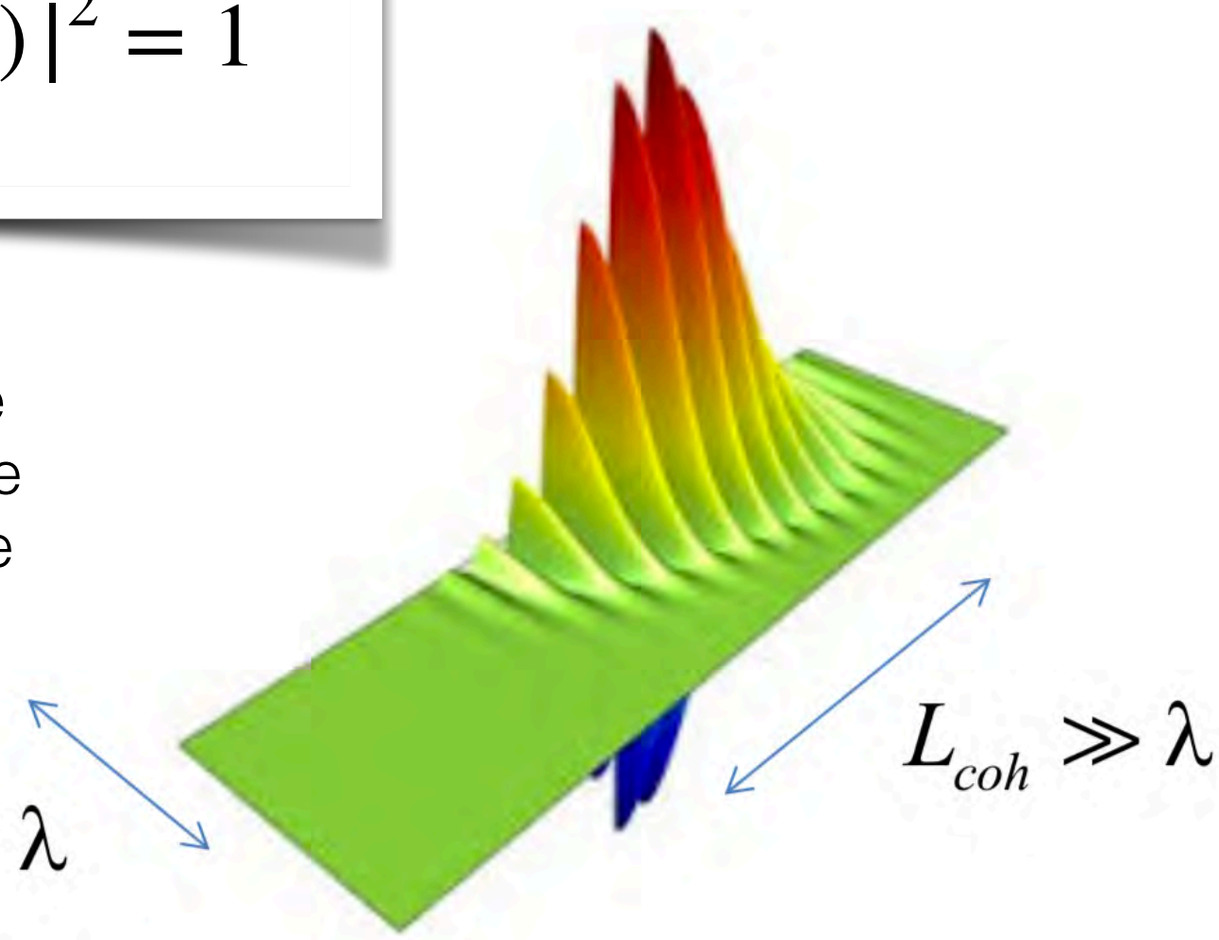
From small-scale photonic chips … to large-scale quantum networks

Single-photon wave packets

$$|\psi\rangle = \int d\nu F(\nu)|\nu\rangle$$

$$|\nu\rangle = a^{\dagger}(\nu)|0\rangle, \quad \int d\nu |F(\nu)|^2 = 1$$

"We note that no restrictions whatever are placed upon the spectral properties of the state $|1\text{phot}\rangle$; any pure one-photon wave packet will do, whatever its frequency distribution may be" [1]

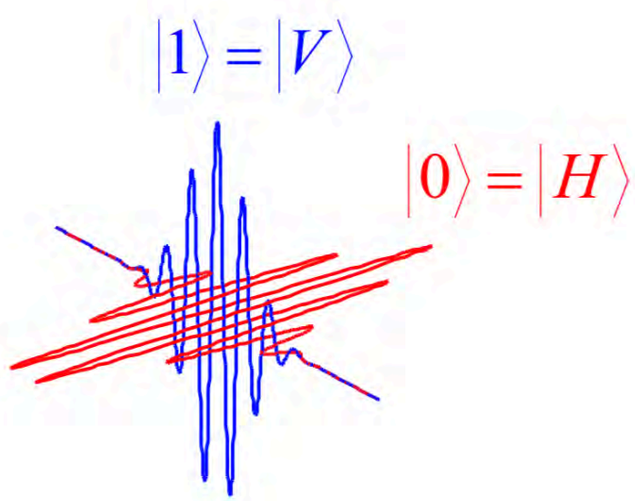


[1] U. M. Titulaer and R. J. Glauber Phys. Rev. 145, 1041 (1966)

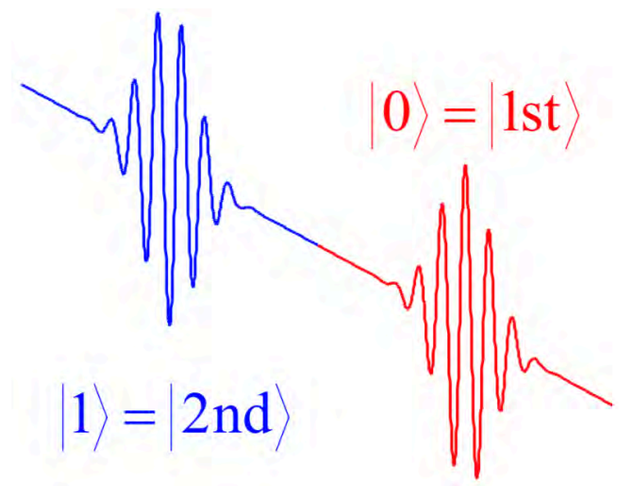
Flying qubits

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

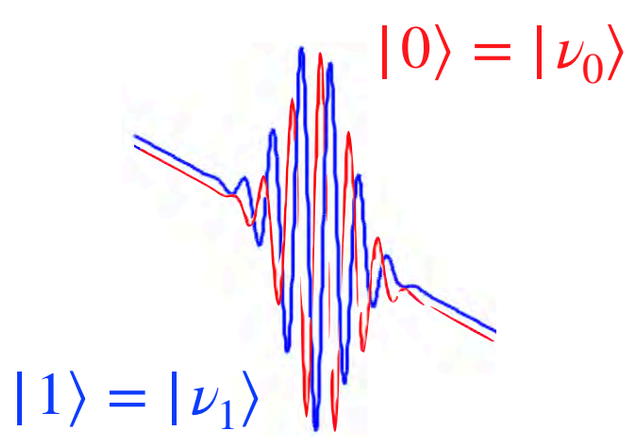
Polarization qubit



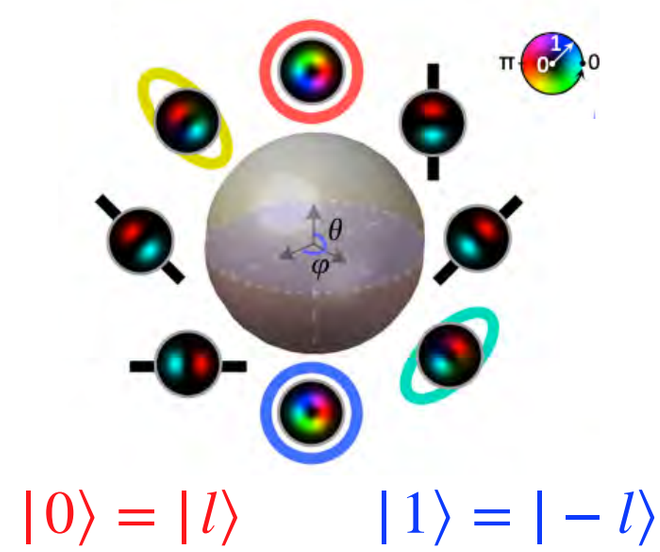
Time-bin qubit



Frequency-bin qubit



OAM-based qubit



High-dimensional quantum bits (qudits)

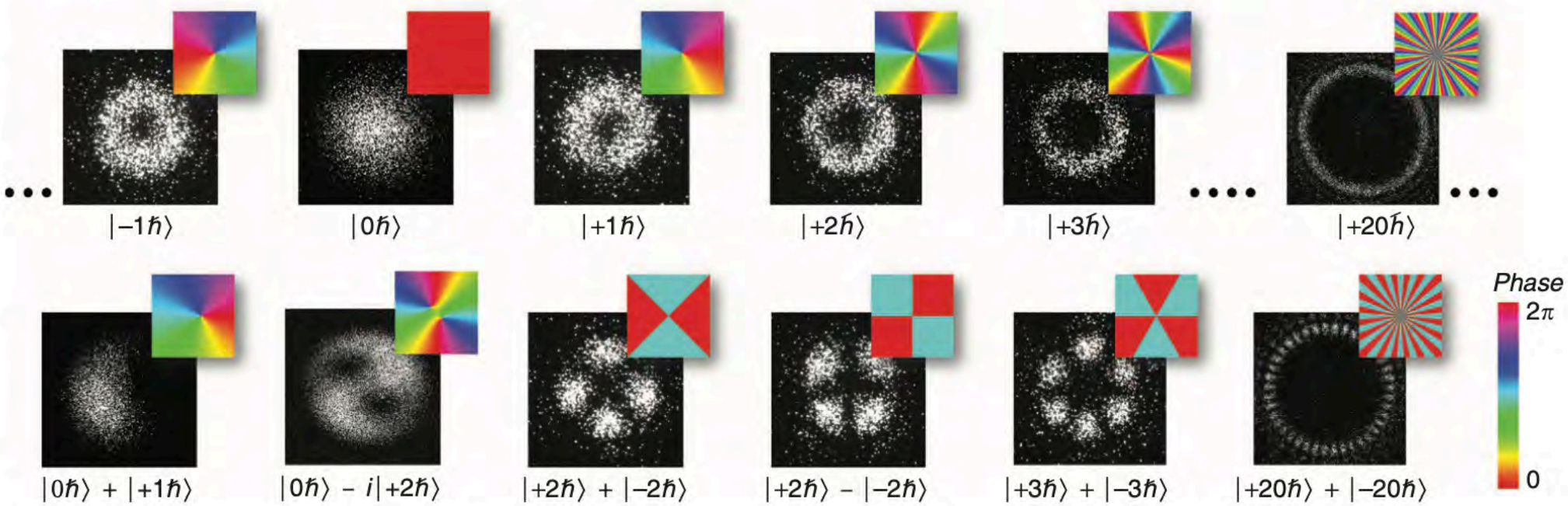


Image: M.Erhard et al. Light: Science & Applications 7, 17146 (2018)

Advantages of qudits:

- Higher information capacity
- Enhanced robustness against eavesdropping and quantum cloning
- Quantum communication without monitoring signal disturbance
- Larger violation of local-realistic theories
- More efficient quantum computation

High-dimensional quantum bits (qudits)

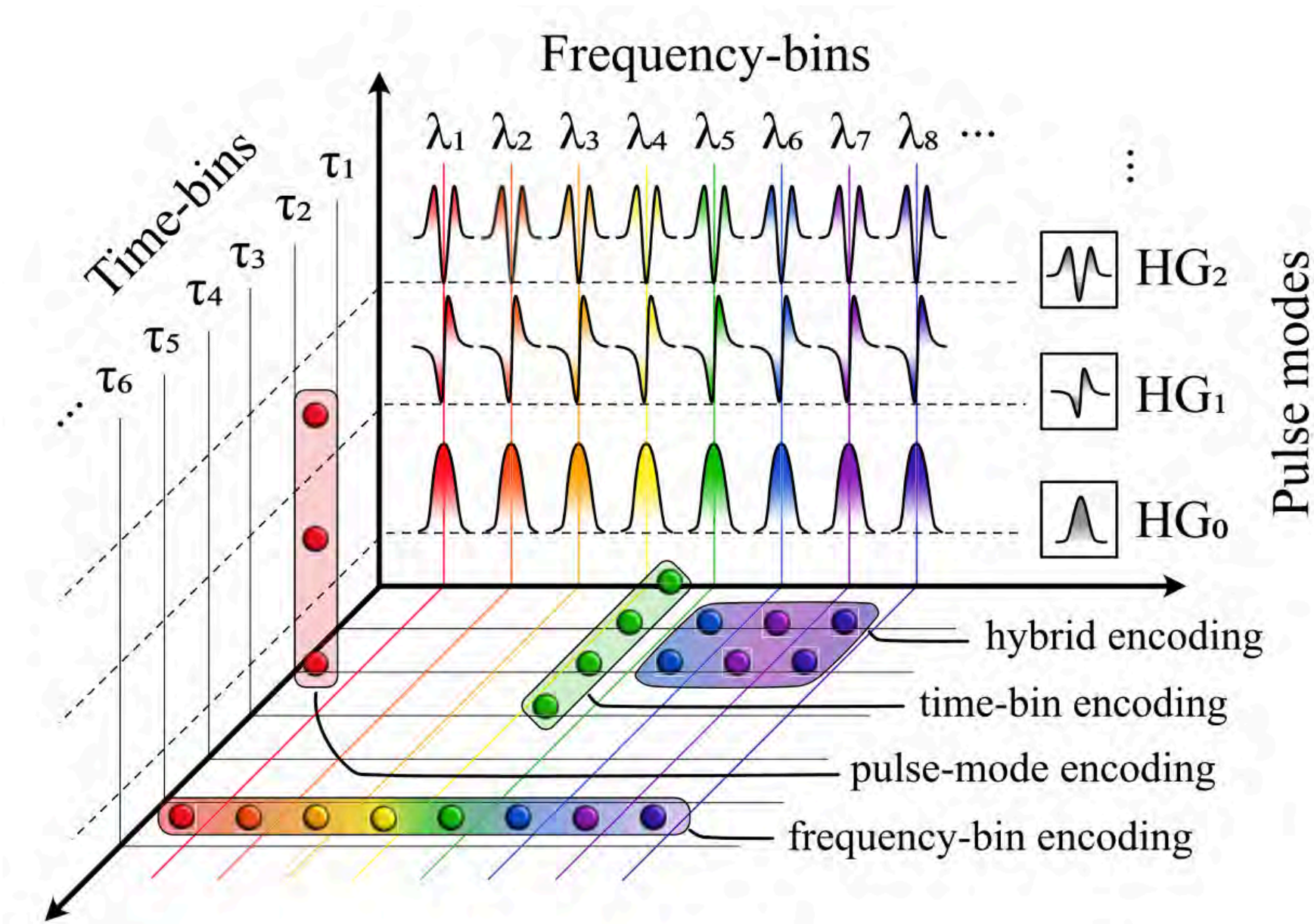


Image: C.L. Morrison et al. APL Photonics 7, 066102 (2022)

High-dimensional quantum bits (qudits)

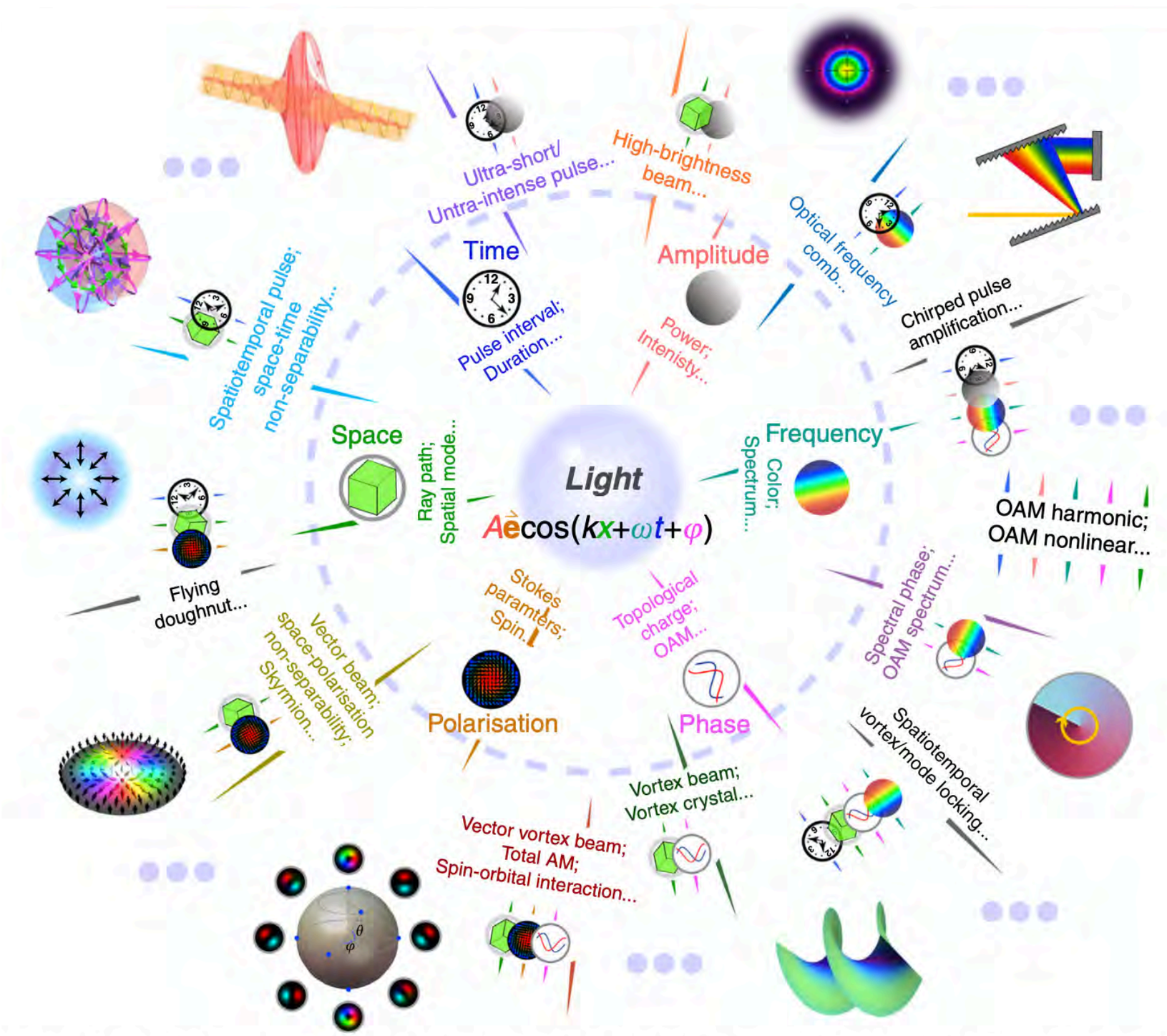


Image: He et al. Light: Science & Applications 11, 205 (2022)

- Introduction
- Heralded single-photon sources
- Single-photon pulse shaping in temporal and spatial domains

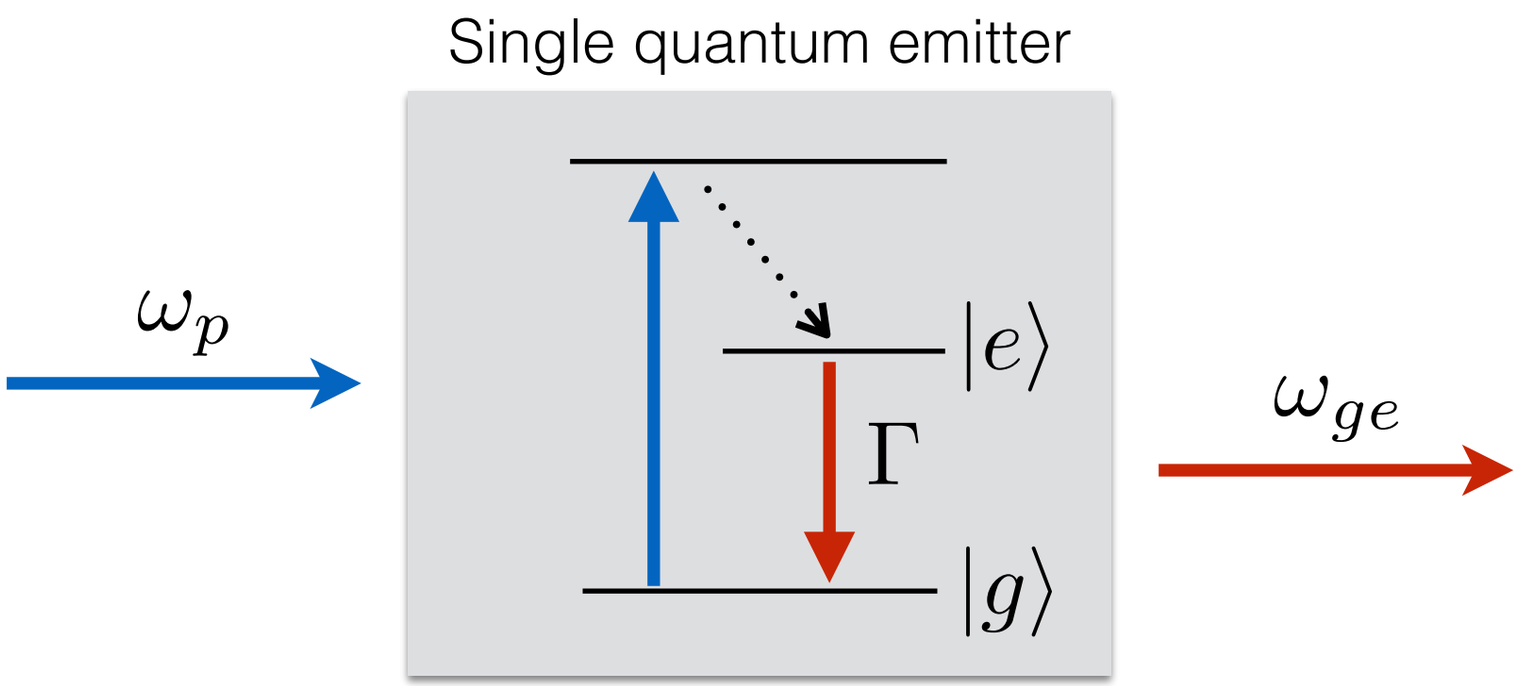
  Optimal schemes maximising heralded efficiency

  Quantum hashing with OAM based qubits
- Heralded sources based on photonic molecules

  Generating pure single-photon states

  Generating frequency-bin qubits
- Conclusion

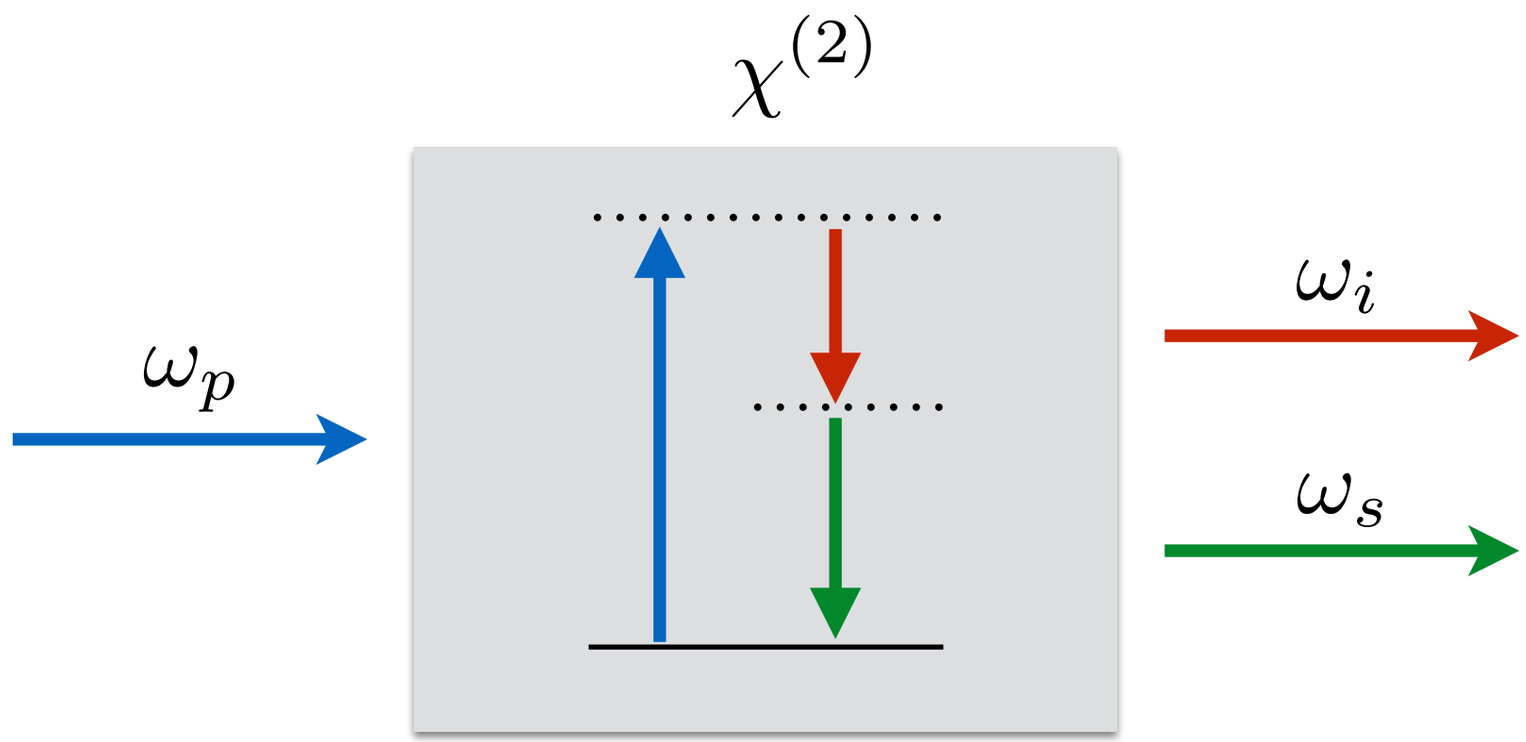
Spontaneous emission



$$|\psi\rangle = \int d\omega \frac{g(\omega)}{(\omega - \omega_{ge}) + i\Gamma/2} |\omega\rangle$$

Single photons on demand

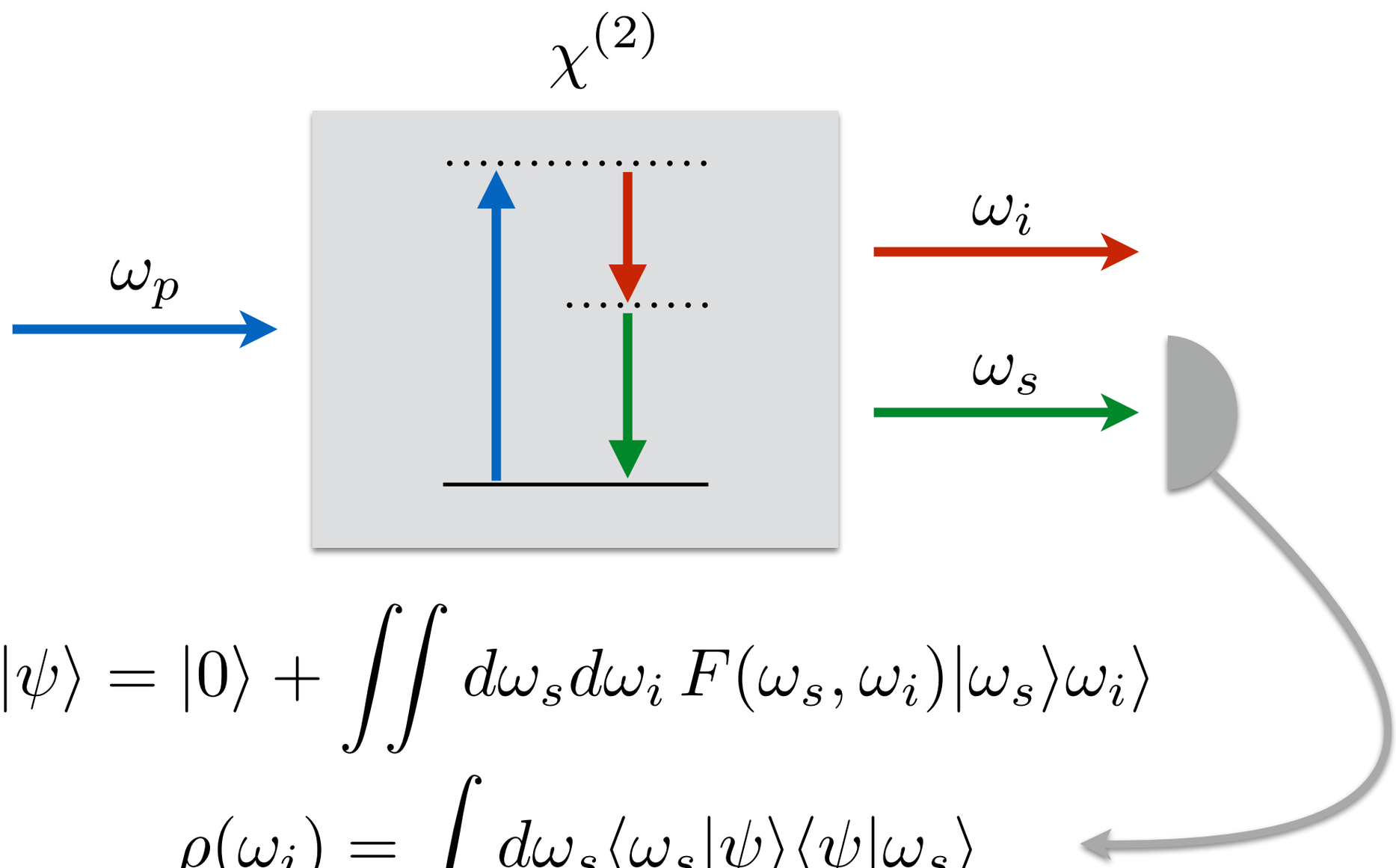
Spontaneous parametric down-conversion



$$|\psi\rangle = |0\rangle + \iint d\omega_s d\omega_i \, F(\omega_s, \omega_i)|\omega_s\rangle\omega_i\rangle$$

Entangled photon pairs

Spontaneous parametric down-conversion

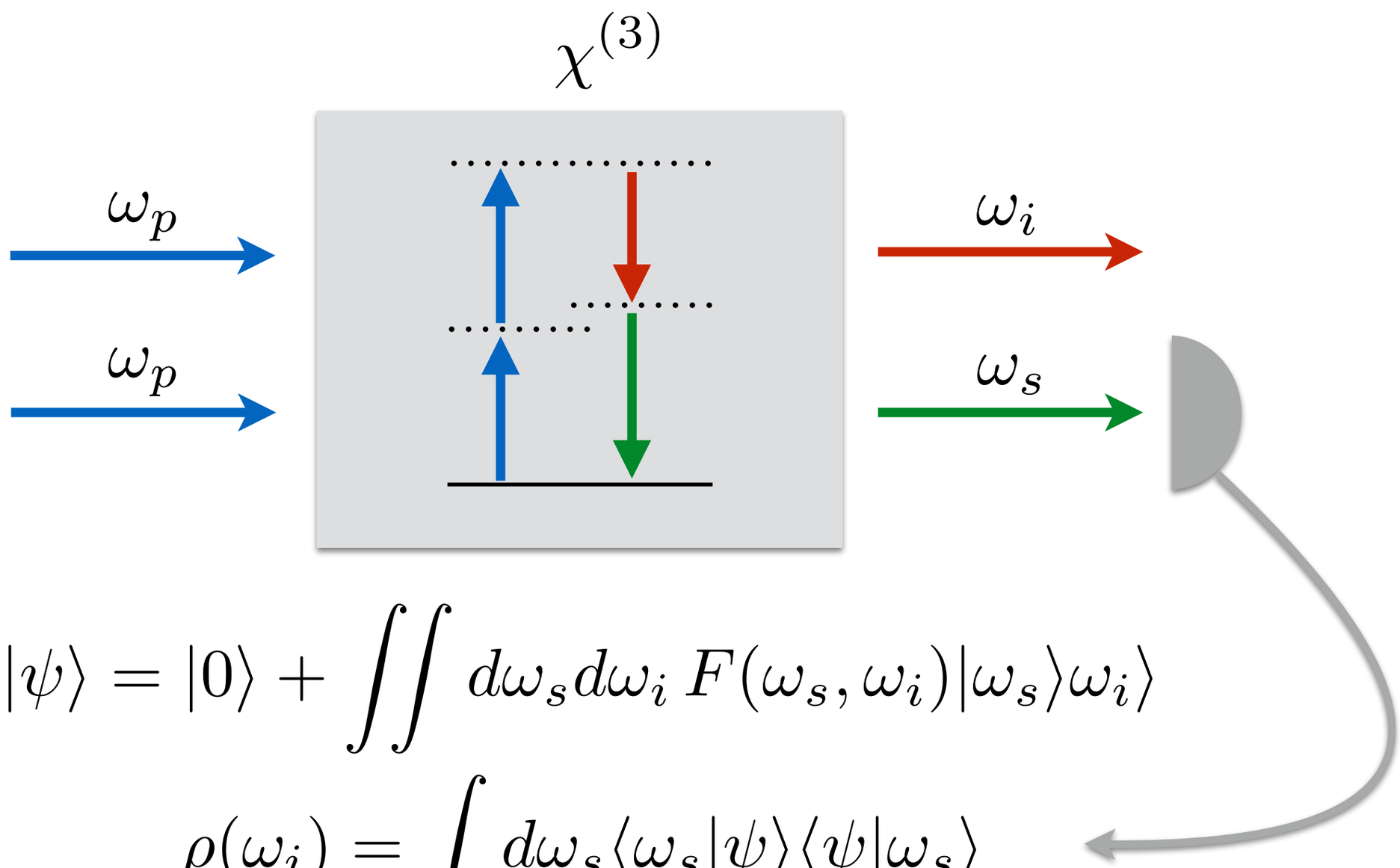


$$|\psi\rangle = |0\rangle + \iint d\omega_s d\omega_i \, F(\omega_s, \omega_i)|\omega_s\rangle\omega_i\rangle$$

$$\rho(\omega_i) = \int d\omega_s \langle\omega_s|\psi\rangle\langle\psi|\omega_s\rangle$$

Entangled photon pairs
Heralded single photons

Spontaneous four-wave mixing

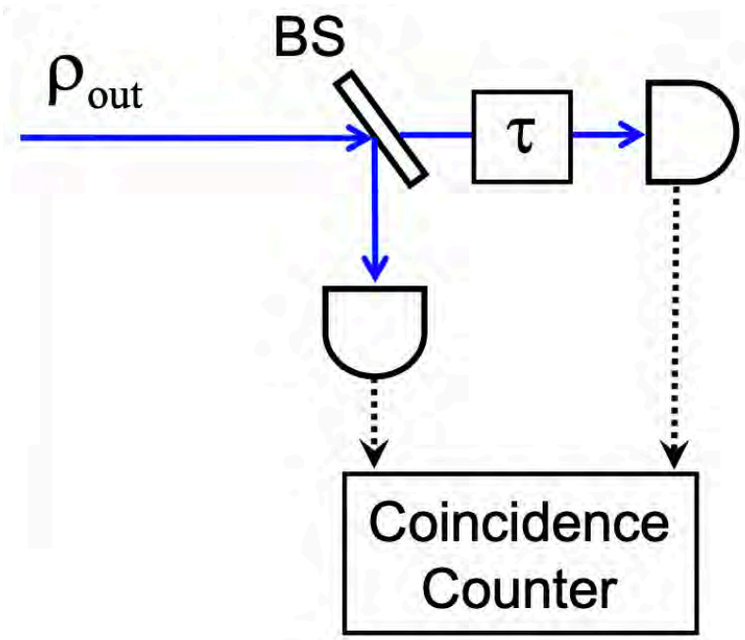


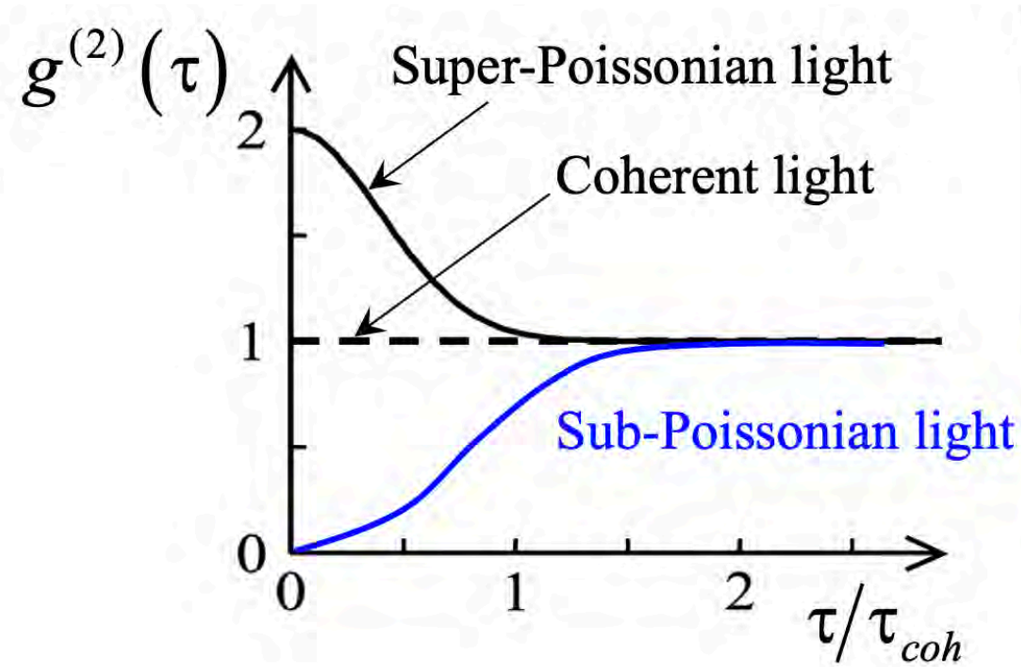
$$|\psi\rangle = |0\rangle + \iint d\omega_s d\omega_i \, F(\omega_s, \omega_i)|\omega_s\rangle\omega_i\rangle$$

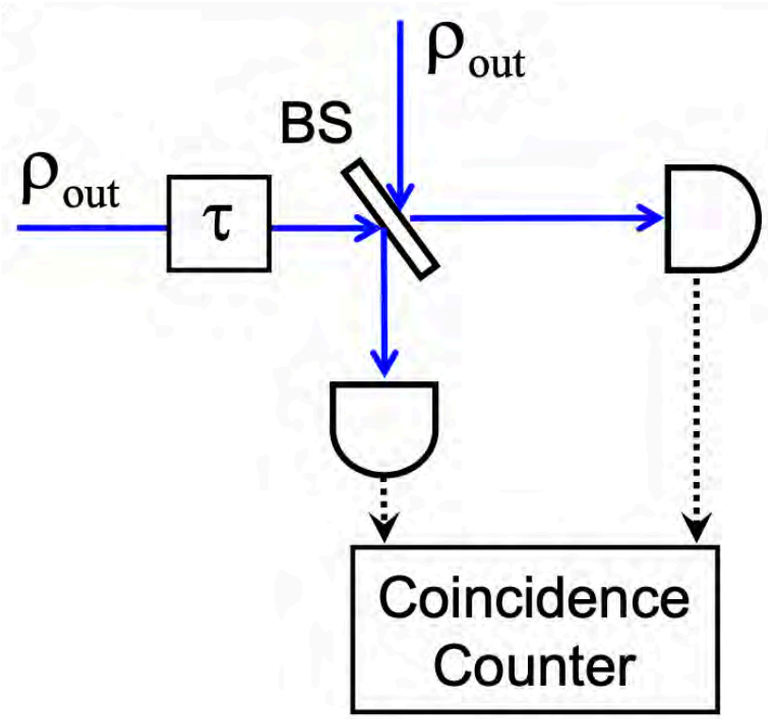
$$\rho(\omega_i) = \int d\omega_s \langle\omega_s|\psi\rangle\langle\psi|\omega_s\rangle$$

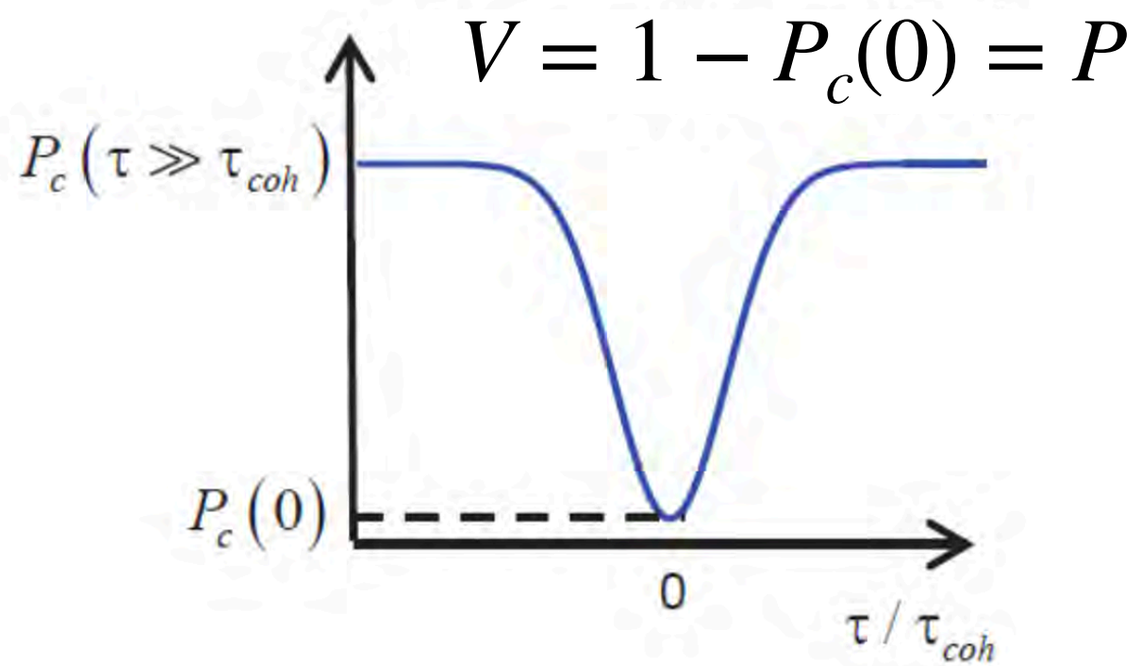
Entangled photon pairs
Heralded single photons

Basic figures of merit

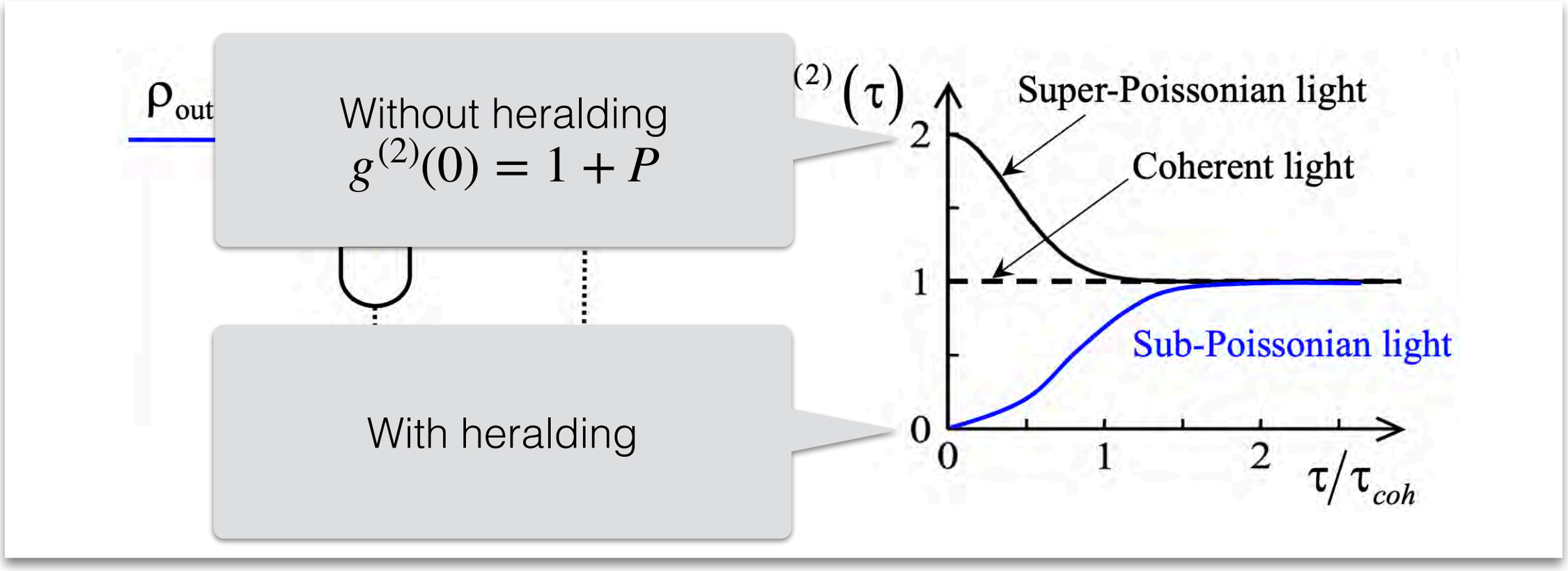


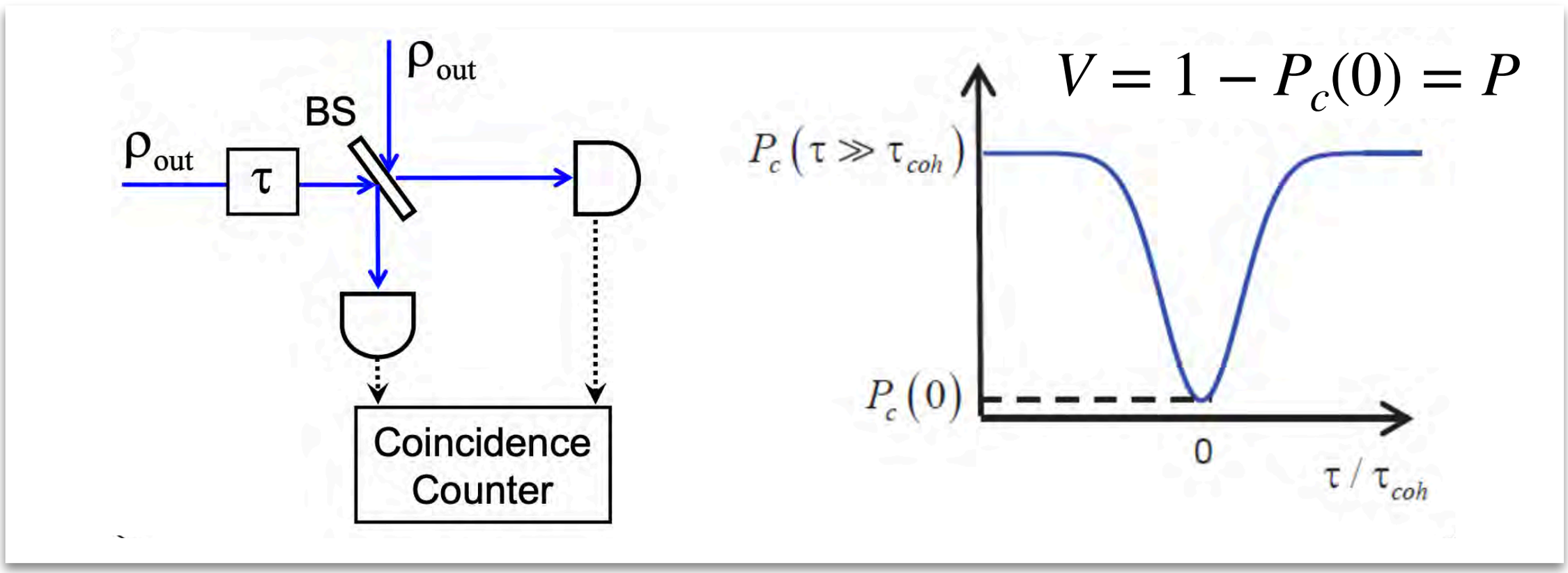






Basic figures of merit





Efficient high purity photon sources

$$|\psi\rangle = |0\rangle + \iint d\omega_s d\omega_i \, F(\omega_s, \omega_i)|\omega_s\rangle\omega_i\rangle$$

$$\rho(\omega_i) = \int d\omega_s \langle\omega_s|\psi\rangle\langle\psi|\omega_s\rangle$$

Factorable JSA

$F(\omega_i, \omega_s) = F_i(\omega_i)F_s(\omega_s)$

⇨

Pure heralded states

$\rho(\omega_i) = |\psi_i\rangle\langle\psi_i|$

A.B.U'Ren, K. Banaszek, I.A. Walmsley // Quantum Inf. Comp. 3, 480 (2003)
A.B.U'Ren, C. Silberhorn, et al. // Laser Physics, 15, 146 (2005)

No manipulation
with heralded photon

⇨

Maximum
heralded efficiency

Comments on JSA

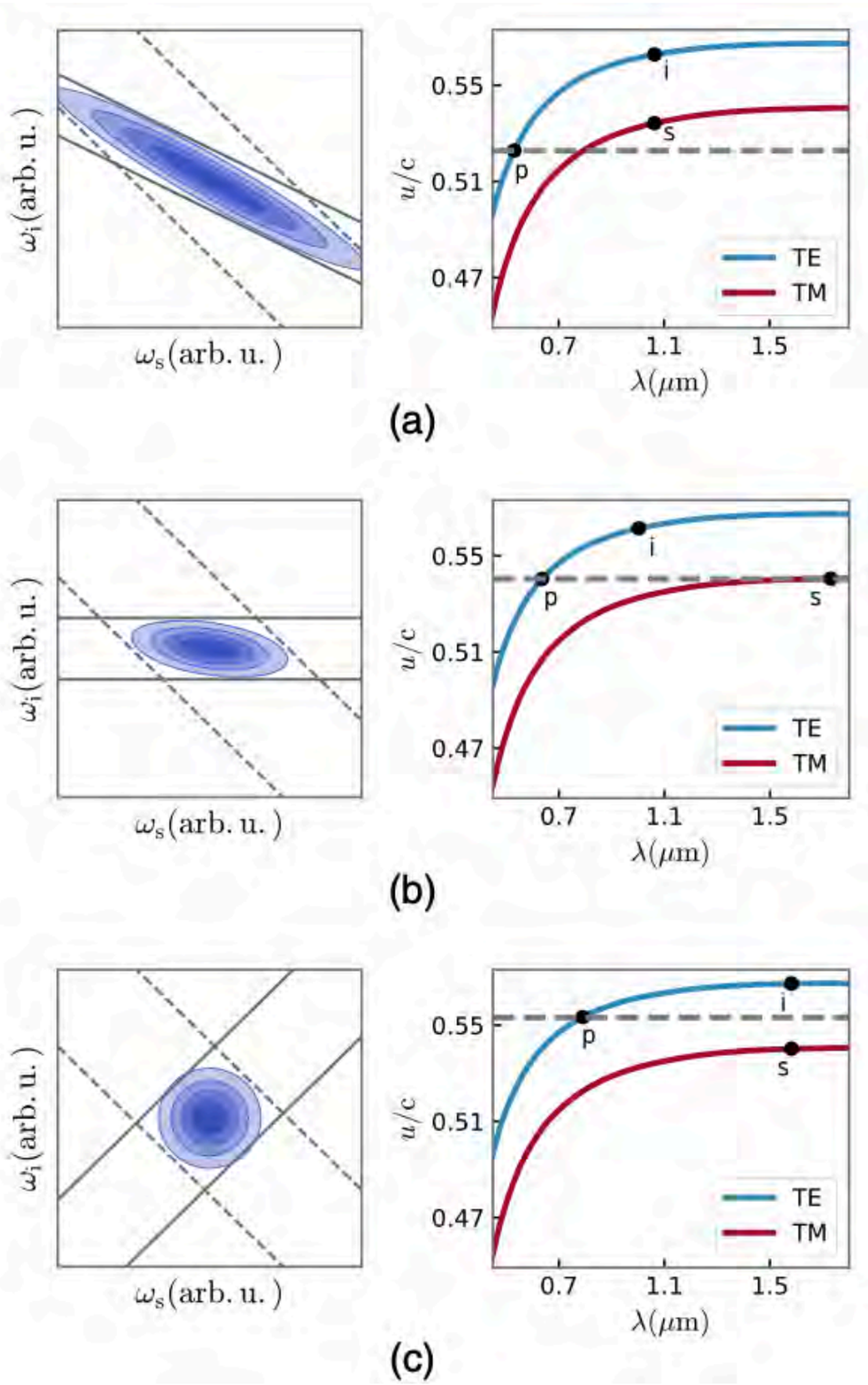


Image: V. Ansari, et al. Optica, 5, 534 (2018)

$$F(\omega_s, \omega_i) = E_p(\omega_s + \omega_i)\phi(\omega_s, \omega_i)$$

$$\phi(\omega_s, \omega_i) \propto \mathrm{sinc}(\Delta k(\omega_s, \omega_i)L/2)$$

$$\Delta k(\omega_s, \omega_i) = (u_s^{-1} - u_p^{-1})\omega_s + (u_i^{-1} - u_p^{-1})\omega_i$$

Asymmetric group-velocity matching

$$F(\omega_s, \omega_i) \approx E_p(\omega_s + \omega_i)\phi(\omega_i)$$

Symmetric group-velocity matching (extended phase matching)

$$F(\omega_s, \omega_i) \approx E_p(\omega_s + \omega_i)\phi(\omega_s - \omega_i)$$

Comments on JSA

Schmidt decomposition:

$$\mathcal{F}(\omega_i, \omega_s) = \sum_n \sqrt{\lambda_n}\, \psi_n(\omega_i)\phi_n(\omega_s), \quad \sum_n \lambda_n = 1$$

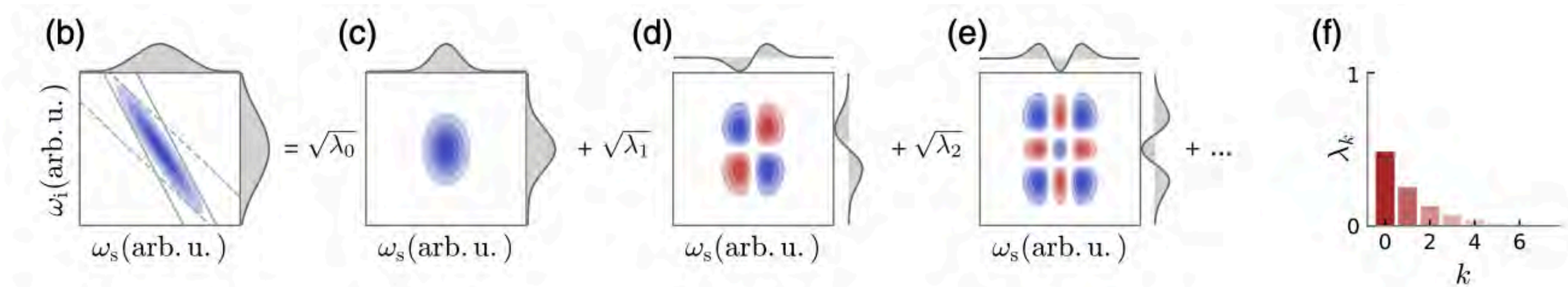


Image: V. Ansari, et al. Optica, 5, 534 (2018)

Schmidt number: $K = 1/\sum_n \lambda_n^2 \quad (K \geq 1)$

Purity: $P = 1/K \quad (0 \leq P \leq 1)$

$K = 1$ → Factorable JSA → Pure state of heralded photons

Comments on JSA

Cavity-enhanced SPDC or SFWM

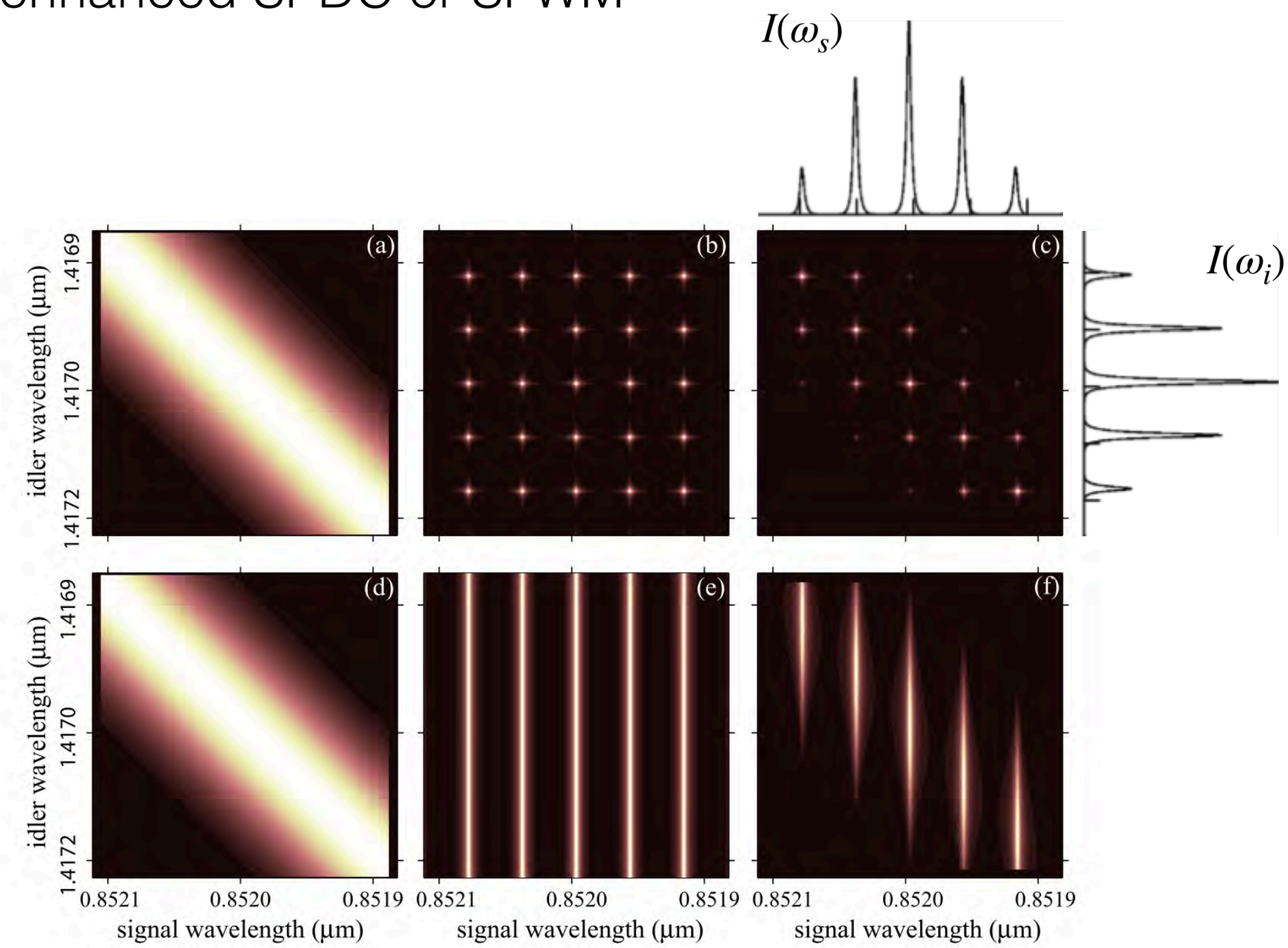


Image: K Garay-Palmett, et al. Laser Physics, 23, 015201 (2013)

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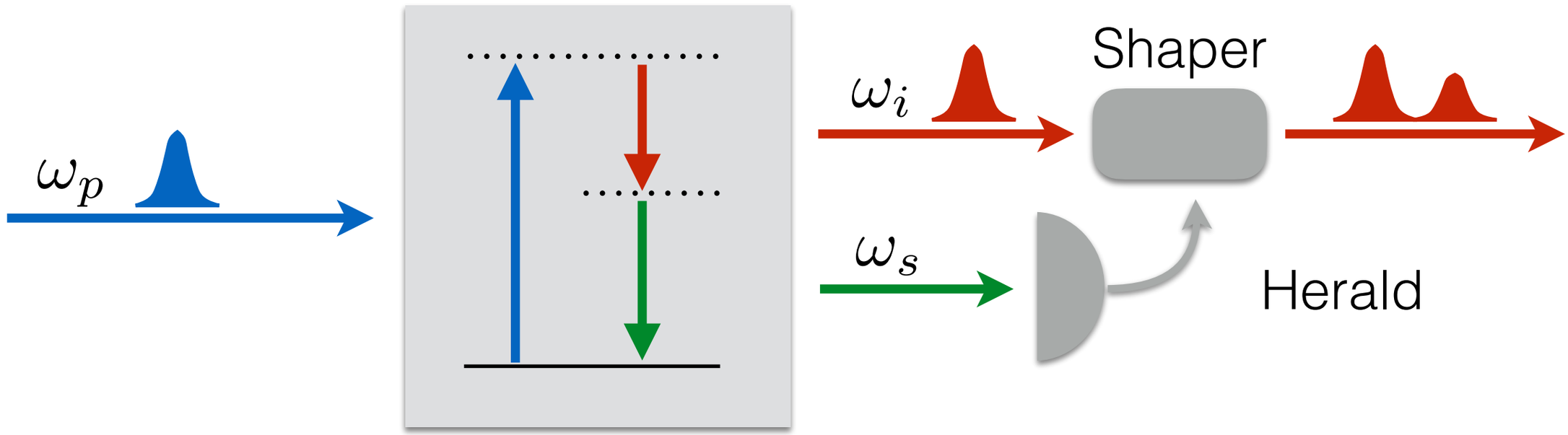
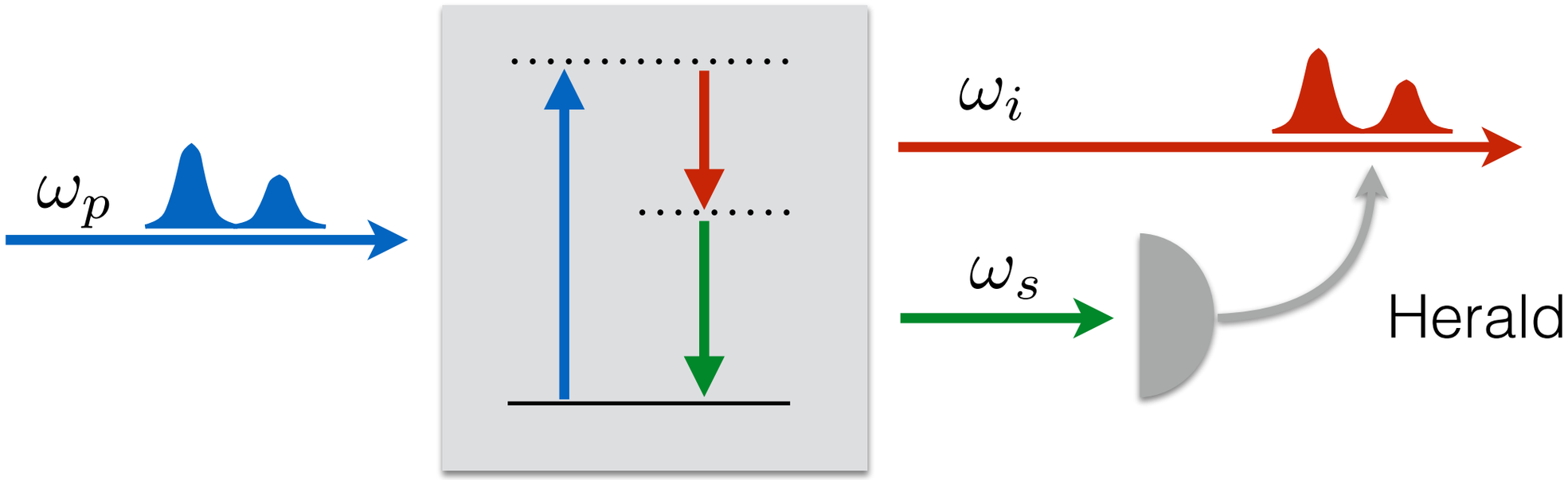
Single-photon pulse shaping: direct approach



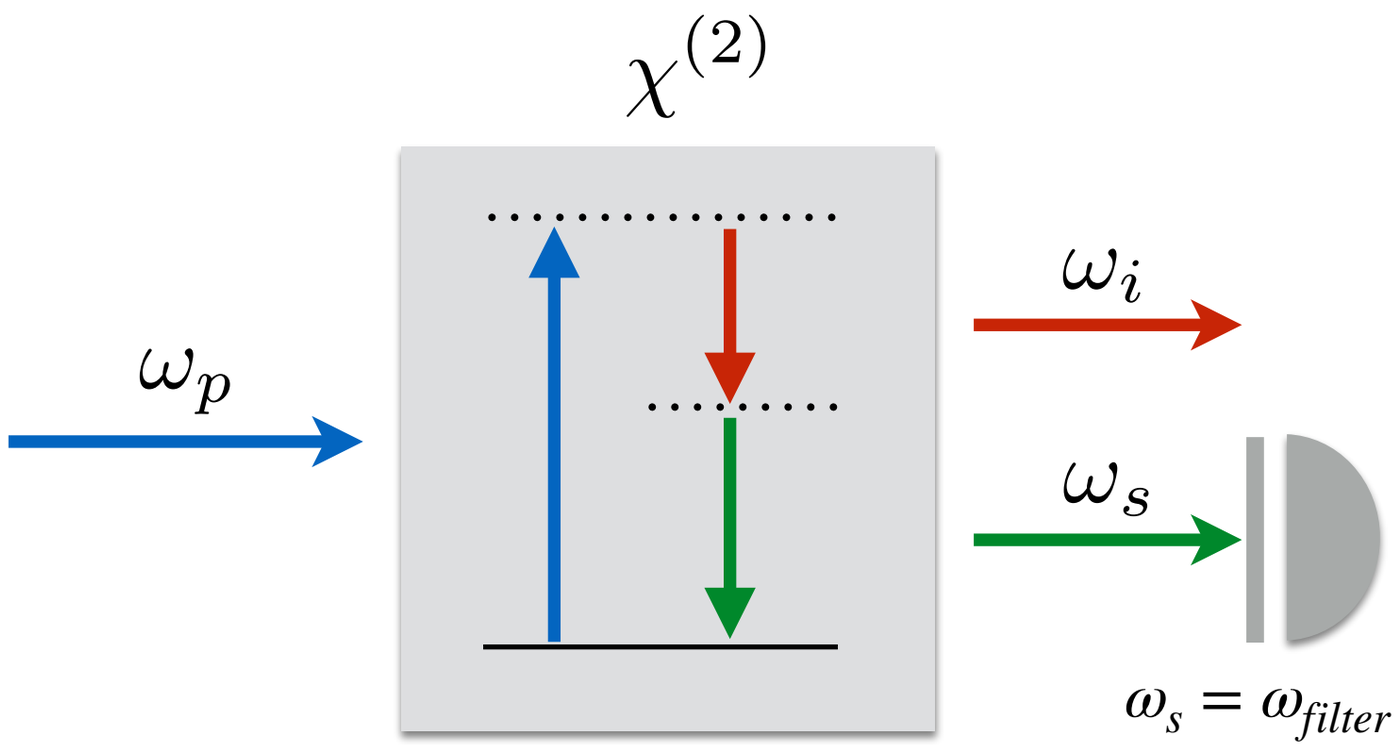
A. Pe'Er, et al. Phys. Rev. Lett. 94, 073601 (2005)
S.-Y. Baek, et al. Phys. Rev. A 77, 013829 (2008)
P. Kolchin, et al. Phys. Rev. Lett. 101, 103601 (2008)

Single-photon pulse shaping via pump field



Z. Y. Ou, Quantum Semiclass. Opt. 9, 599 (1997)
A.Kalachev. PRA, 81, 043809 (2010)
V. Ansari, et al. Optics Express, 26, 2764 (2018)

Single-photon pulse shaping via pump field



$$\omega_p = \omega_i + \omega_s$$

$$\omega_s = \omega_{filter} \rightarrow \omega_i \propto \omega_p$$

$$|\psi_p\rangle = \sum_j \alpha_j |\omega_j\rangle$$

$$\downarrow$$

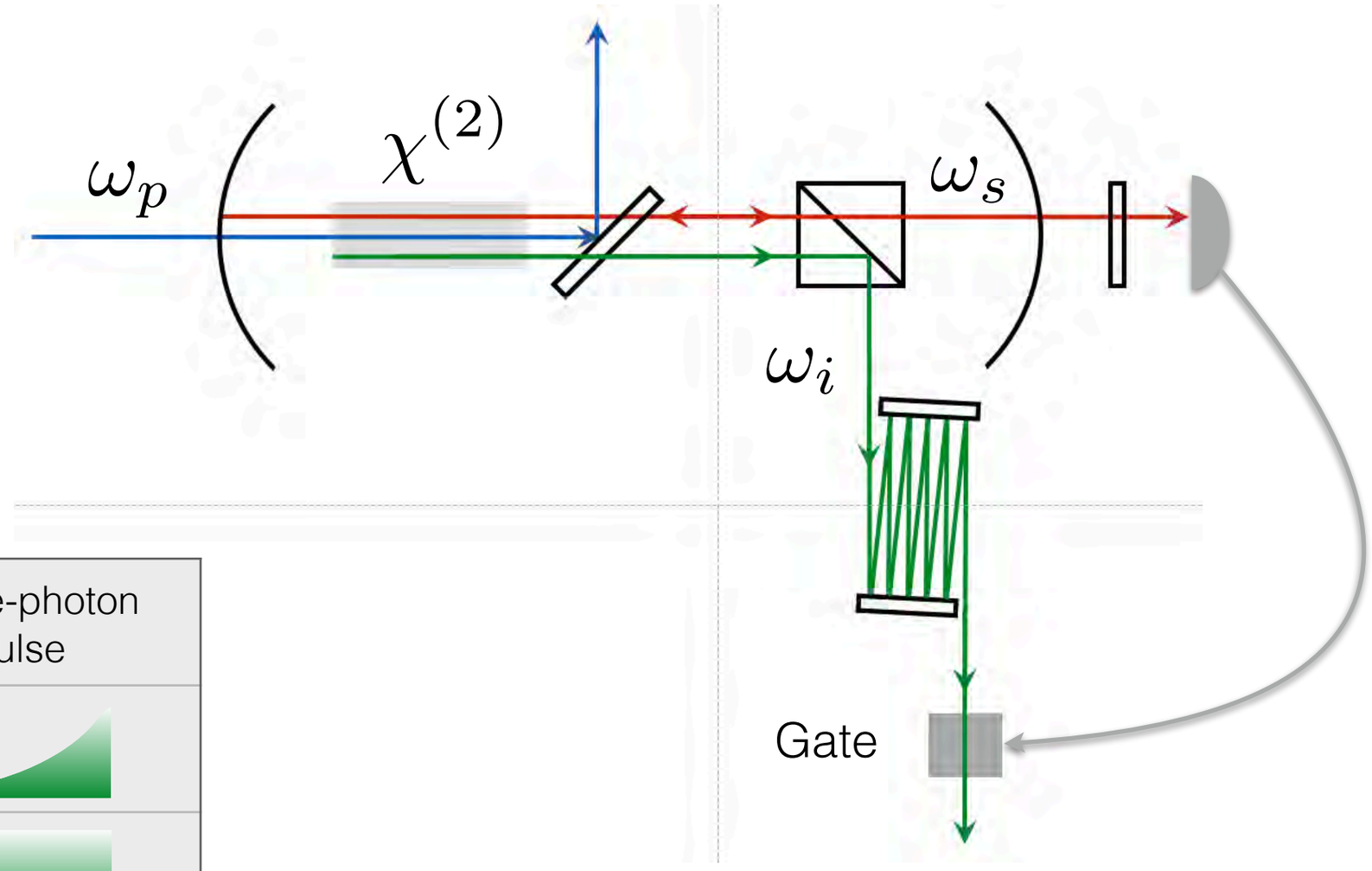
$$|\psi_i\rangle = \sum_j \alpha_j |\omega_j\rangle$$

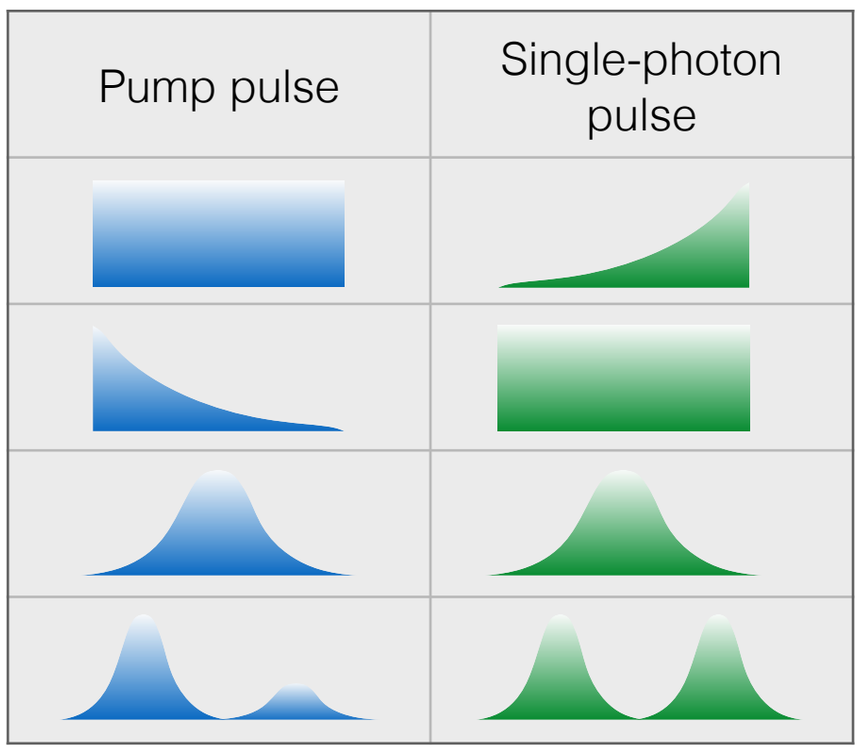
Z. Y. Ou, Quantum Semiclass. Opt. 9, 599 (1997)

Single-photon pulse shaping via pump field

$$E_p(t) = F(t)\exp\left(-\frac{\Gamma}{2}t\right)$$

$$0 \leqslant t \leqslant T$$

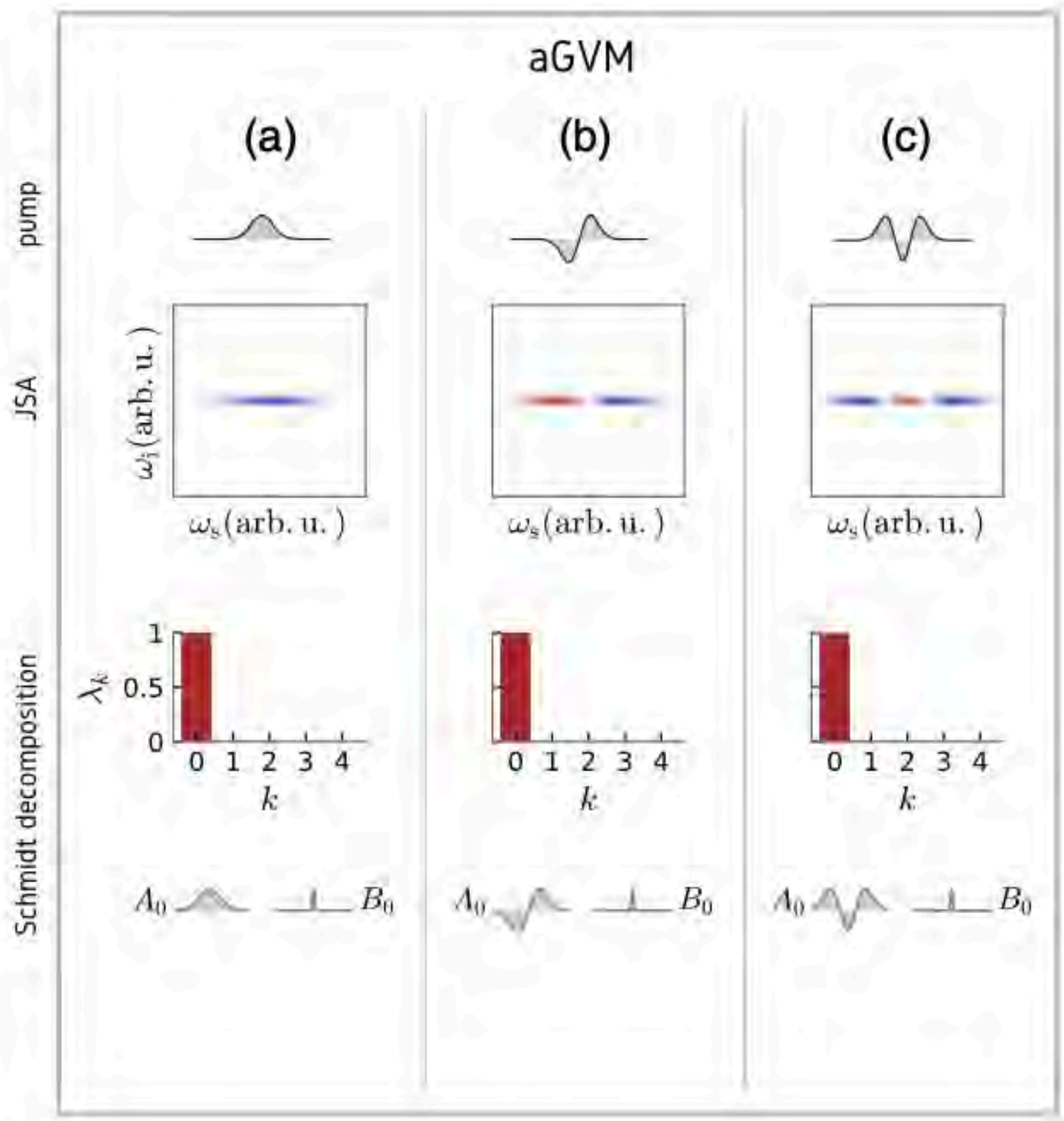




Qubits and qudits in the temporal domain

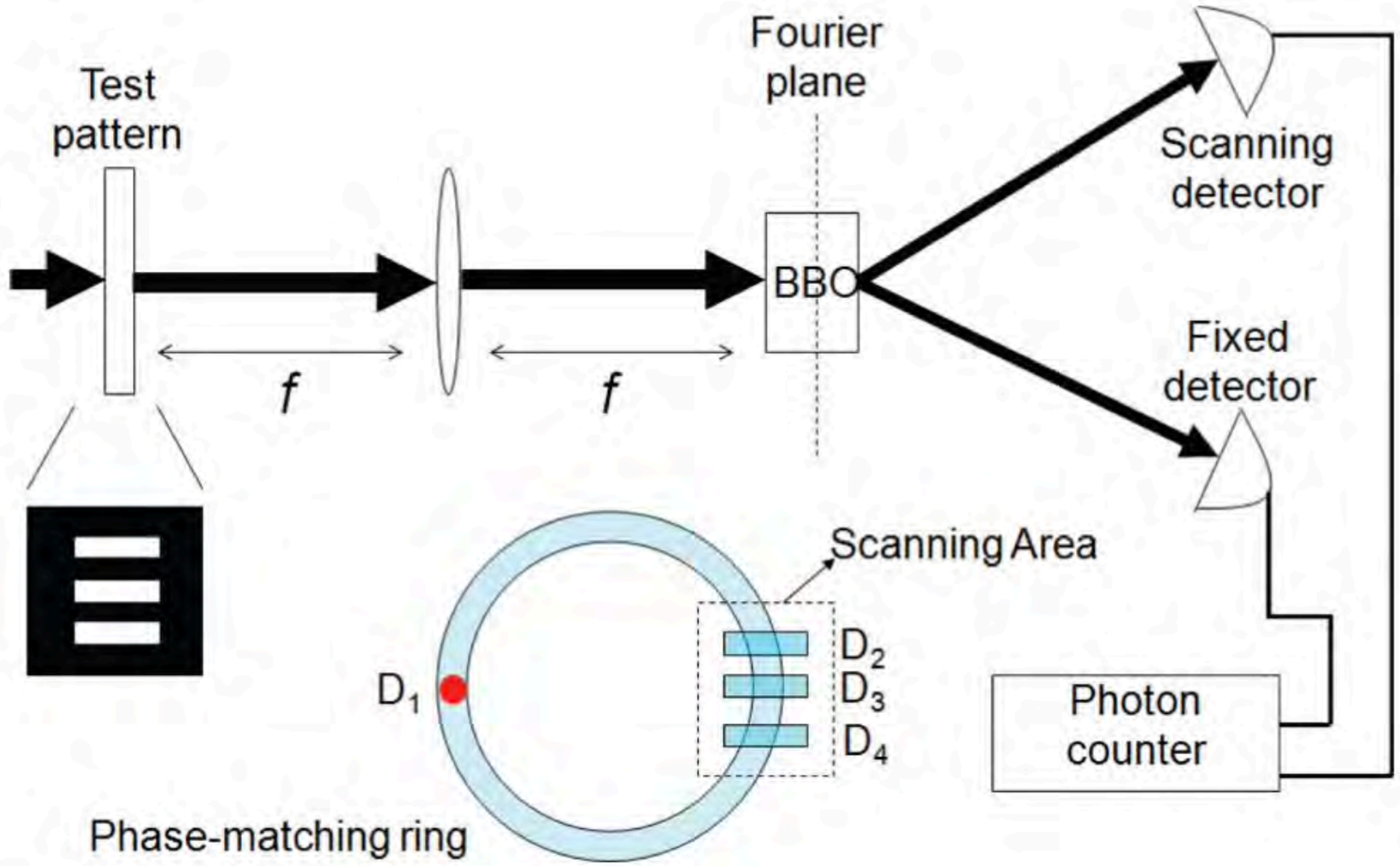
A.Kalachev. PRA, 81, 043809 (2010)

Single-photon pulse shaping via pump field

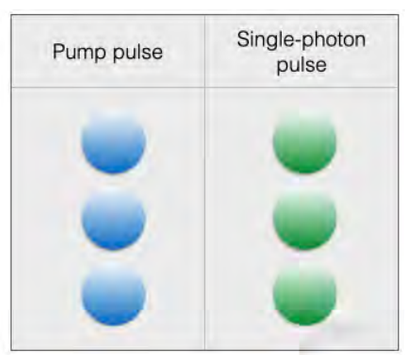


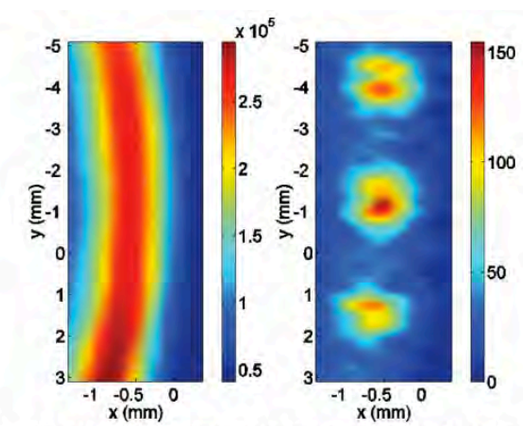
V. Ansari, et al. Optica, 5, 534 (2018)

Single-photon pulse shaping via pump field



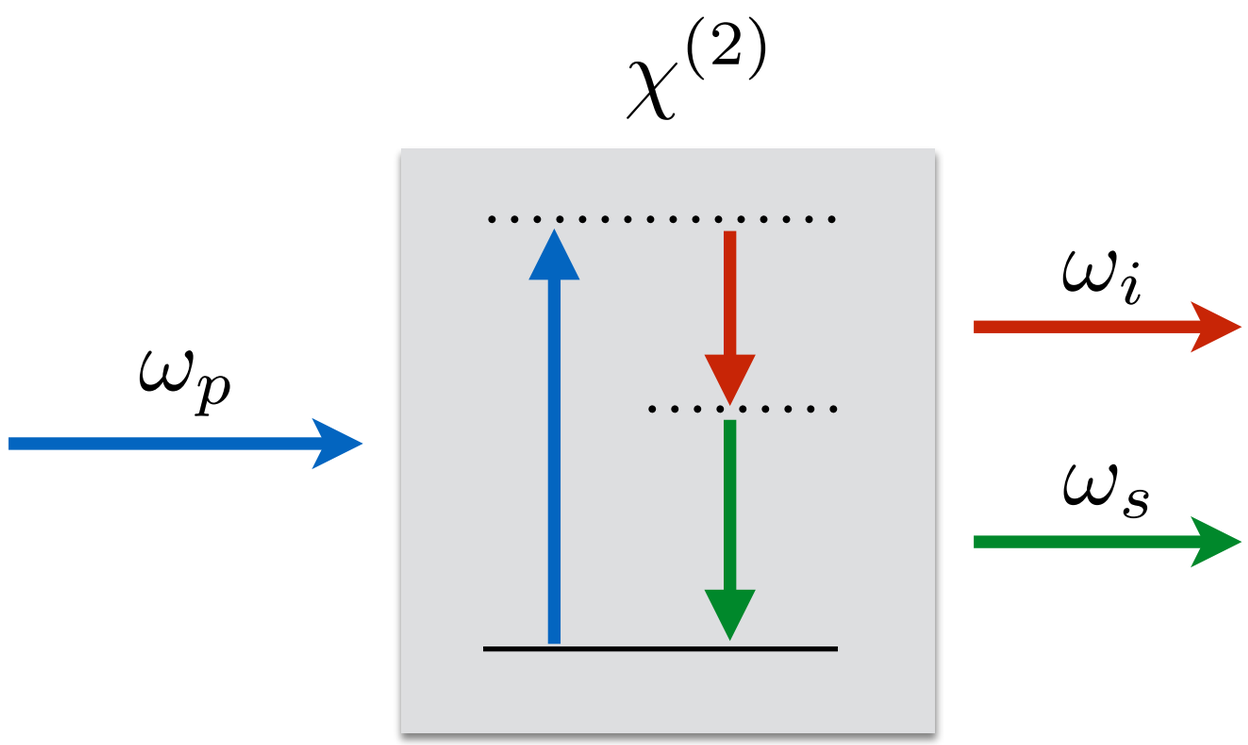
$$\omega_p = \omega_i + \omega_s$$
$$\vec{k}_p = \vec{k}_i + \vec{k}_s$$





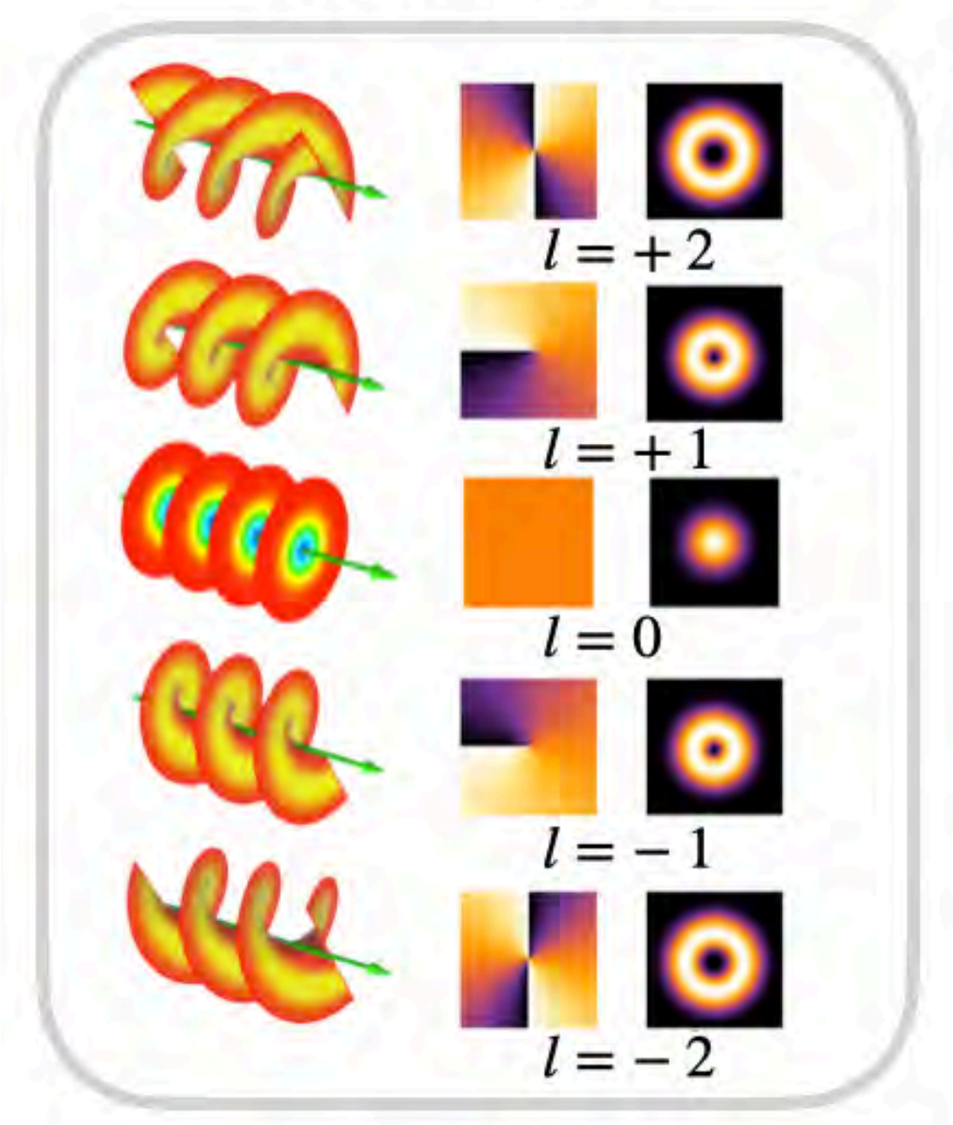
K.G. Köprülü, et al. OL, 36, 1674 (2011)

Generating photonic qubits in OAM basis



$$\omega_p = \omega_i + \omega_s$$

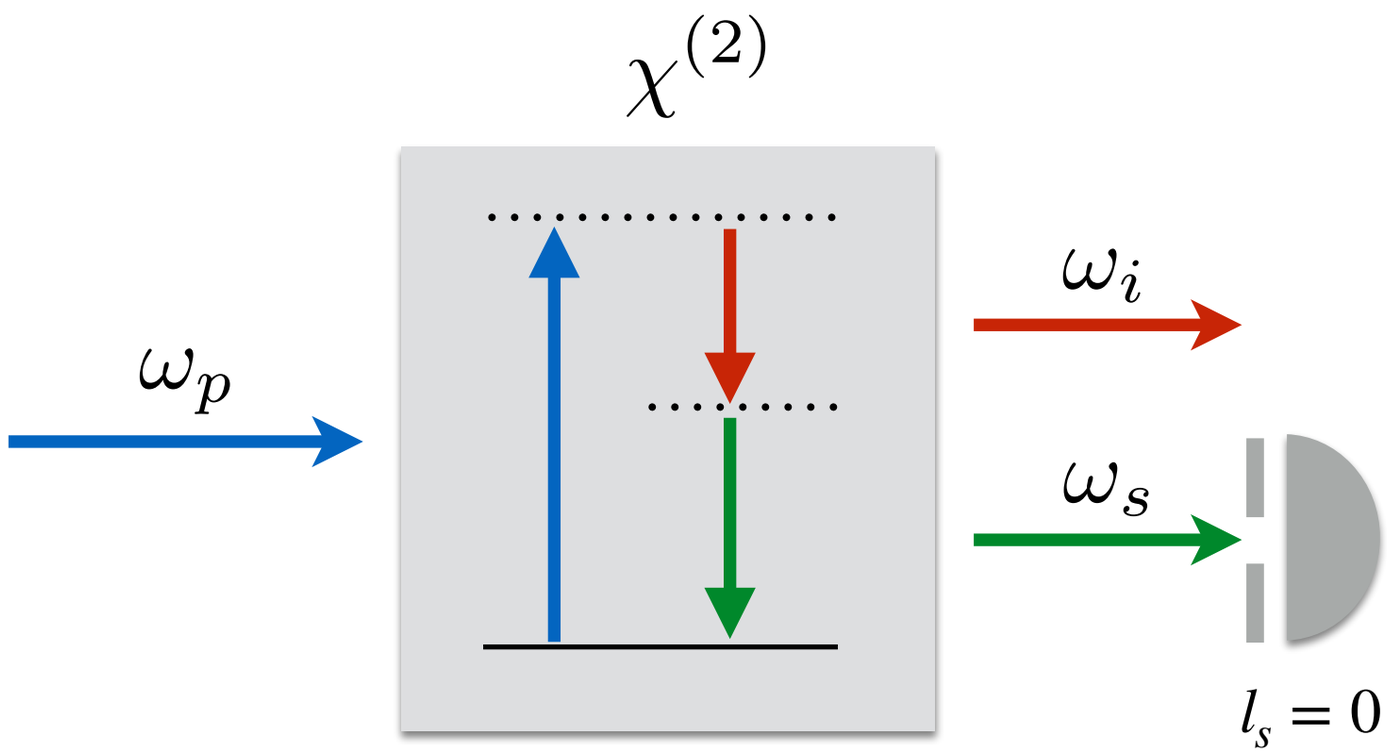
$$l_p = l_i + l_s$$



From Wikipedia

$$\mathrm{LG}_p^l(\rho, \phi, z) \sim \exp(il\phi)$$

Generating photonic qudits in OAM basis



$$\omega_p = \omega_i + \omega_s$$

$$l_p = l_i + l_s$$

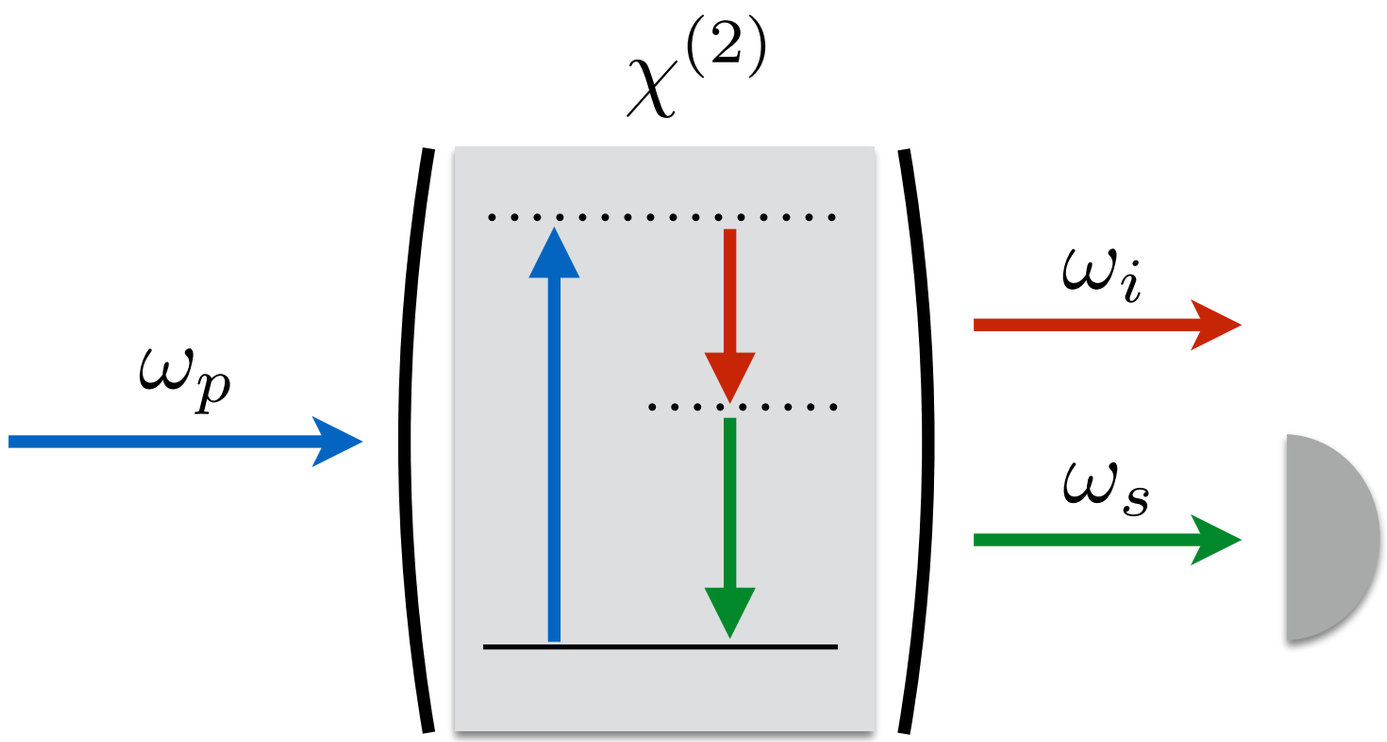
$$l_s = 0 \rightarrow l_i = l_p$$

$$|\psi_p\rangle = \sum_j \alpha_j |l_j\rangle$$

$$\downarrow$$

$$|\psi_i\rangle = \sum_j \alpha_j |l_j\rangle$$

Generating photonic qudits in OAM basis



Single-resonant OPO supporting signal field with $l_s = 0$

$$l_s = 0 \rightarrow l_i = l_p$$

$$\omega_s = \omega_{cav} \rightarrow \omega_i \propto \omega_p$$

$$|\psi_p\rangle = \sum_j \alpha_j |l_j, \omega_j\rangle$$

$$\downarrow$$

$$|\psi_i\rangle = \sum_j \alpha_j |l_j, \omega_j\rangle$$

- Maximum efficiency
- Spatial-temporal or spatial-frequency encoding

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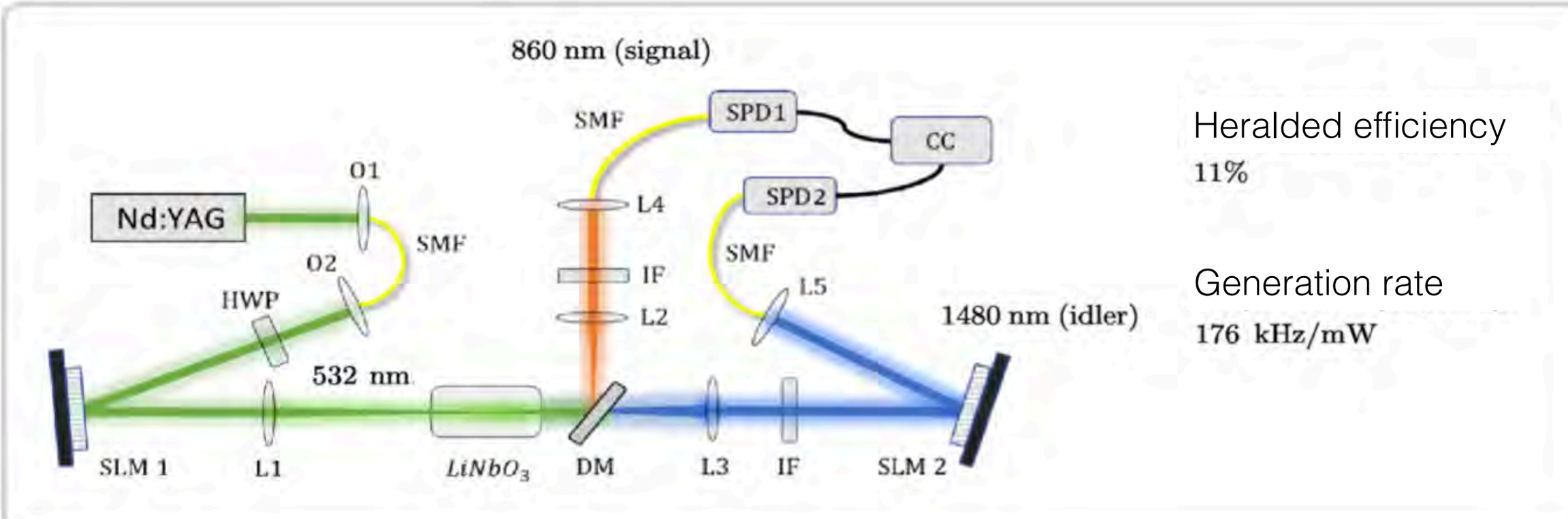
  Generating frequency-bin qubits
- Conclusion



Heralded efficiency

11%

Generation rate

176 kHz/mW

Single-photon qubit states used for quantum hashing

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\ell\rangle + e^{i\varphi}|-\ell\rangle)$$

Accuracy: $\delta\varphi = \pi/256$

Phys. Rev. A, 104, 052606 (2021)

Generating photonic qubits in OAM basis

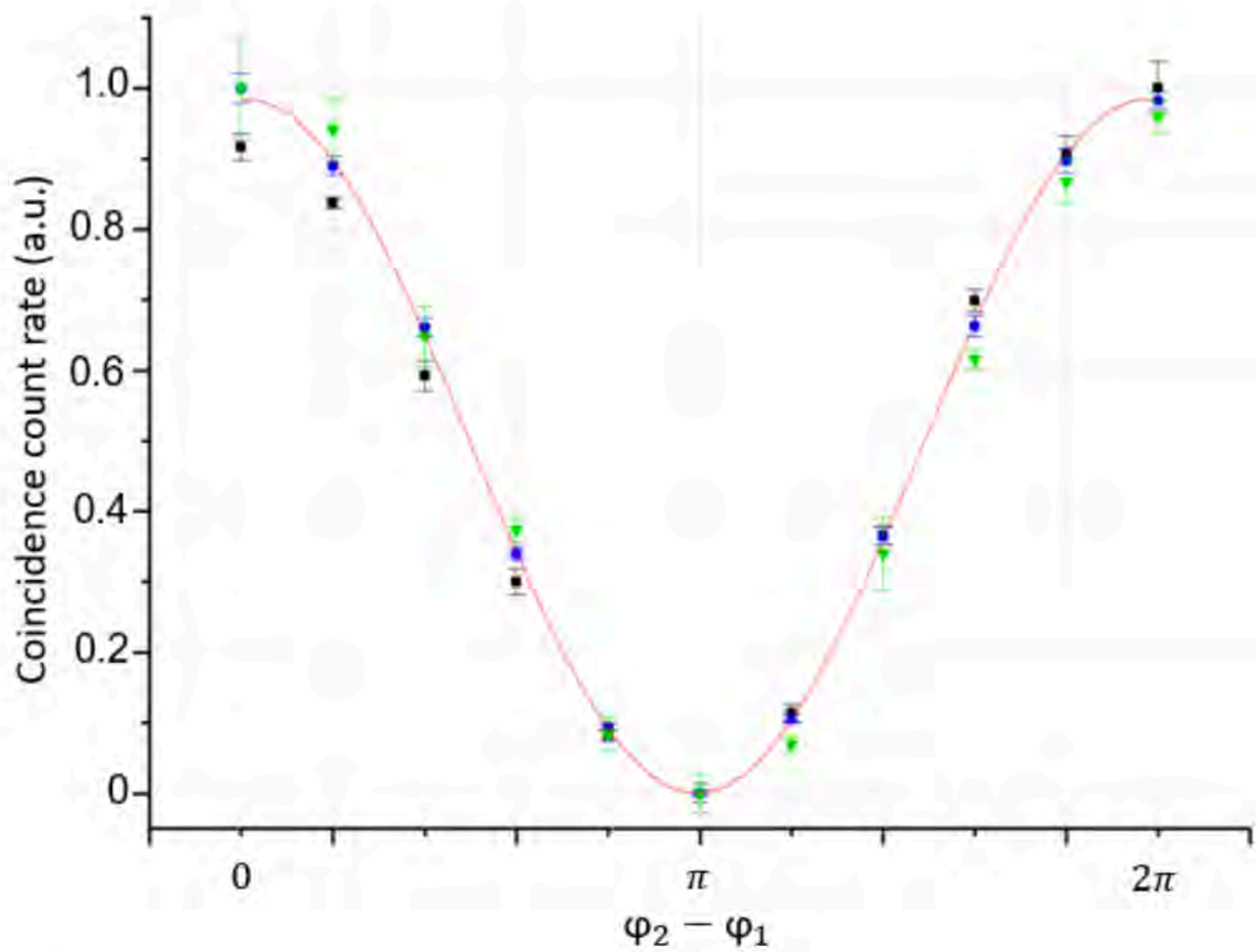


FIG. 2. Coincidence count rate as a function of the phase difference $\varphi_2 - \varphi_1$, where φ_1 corresponds to the qubit state prepared by SLM 1 and φ_2 corresponds to the qubit state for which phase flattering is achieved by SLM 2. Solid line is the theoretical dependence; black square, blue circles, green triangles correspond to $l = 1, 2$ and 3, respectively.

Phys. Rev. A, 104, 052606 (2021)

Quantum hashing

Classical input:

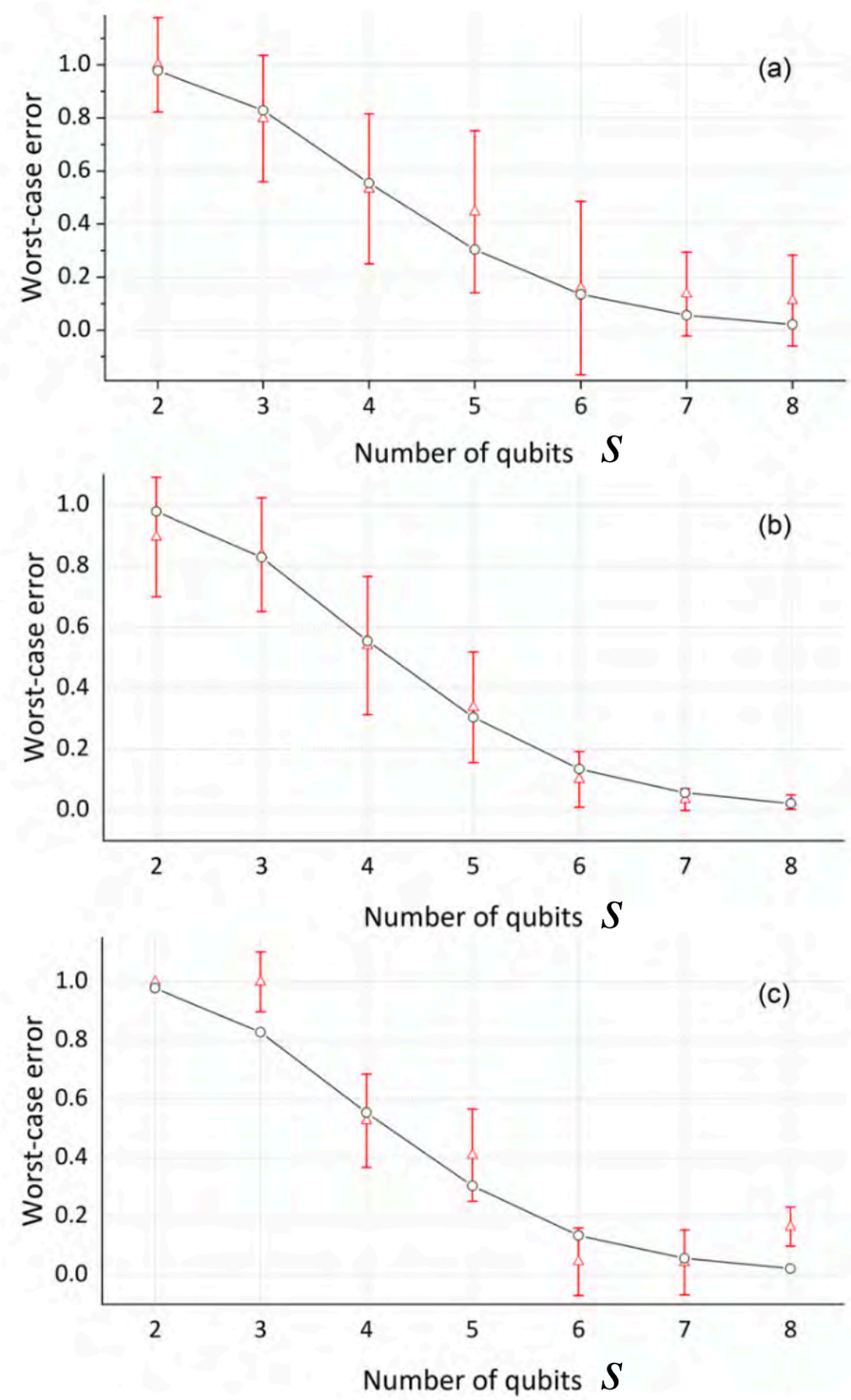
$$x \in \{0, 1, \ldots, q-1\}$$

Quantum hash:

$$|\psi_j(x)\rangle = \frac{1}{\sqrt{2}}\left(|\ell\rangle + e^{i\frac{2\pi b_j x}{q}}|-\ell\rangle\right)$$

$$|\psi(x)\rangle = |\psi_1(x)\rangle \otimes \cdots \otimes |\psi_s(x)\rangle$$

Main requirements are **resistance to inversion** (known as "one-way property"), which makes it unlikely to "extract" encoded information out of the quantum state, and **resistance to quantum collisions**, which means that high similarity of quantum images for different inputs is hardly likely.



$l = \pm 1$

$l = \pm 2$

$l = \pm 3$

$q = 512$ (9 bits)

Phys. Rev. A, 104, 052606 (2021)

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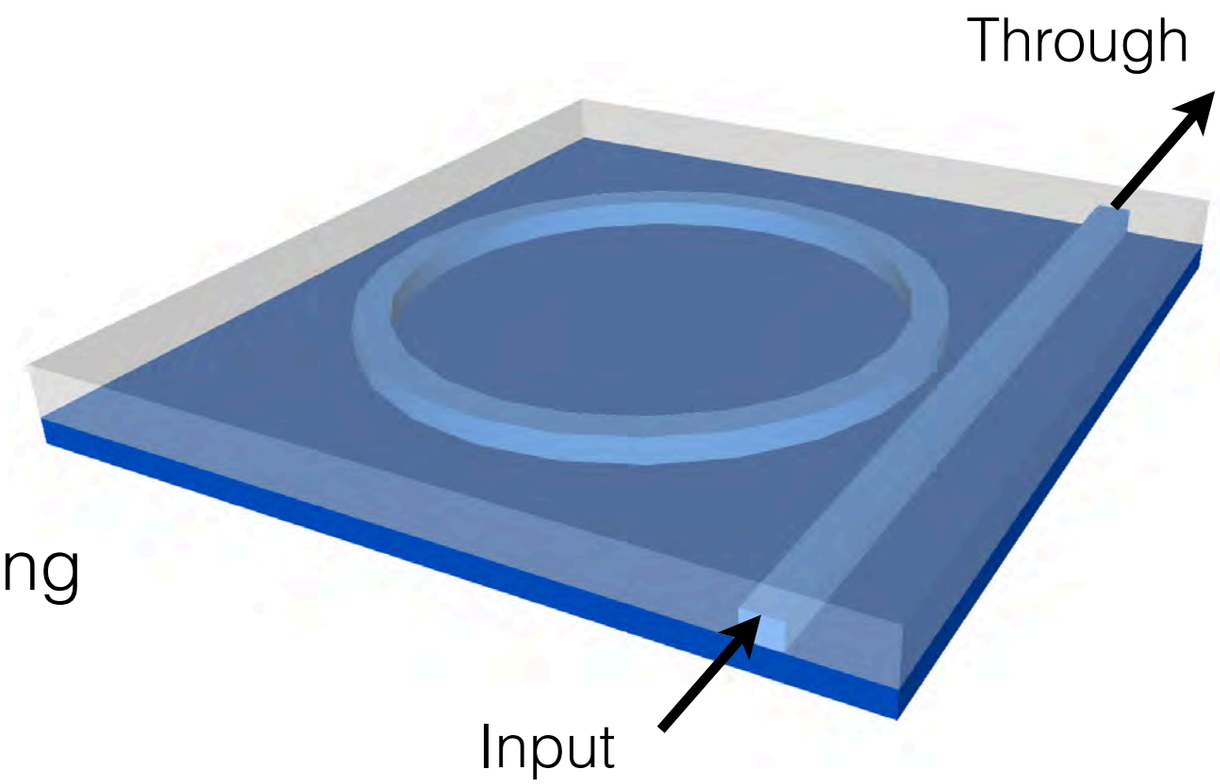
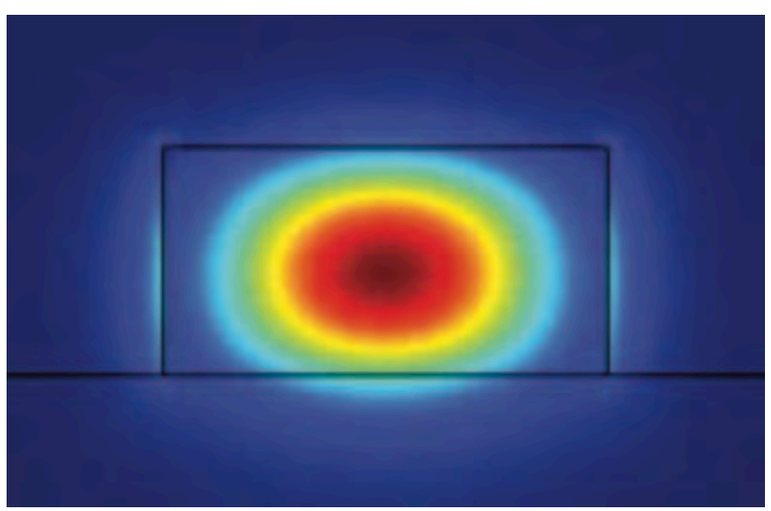
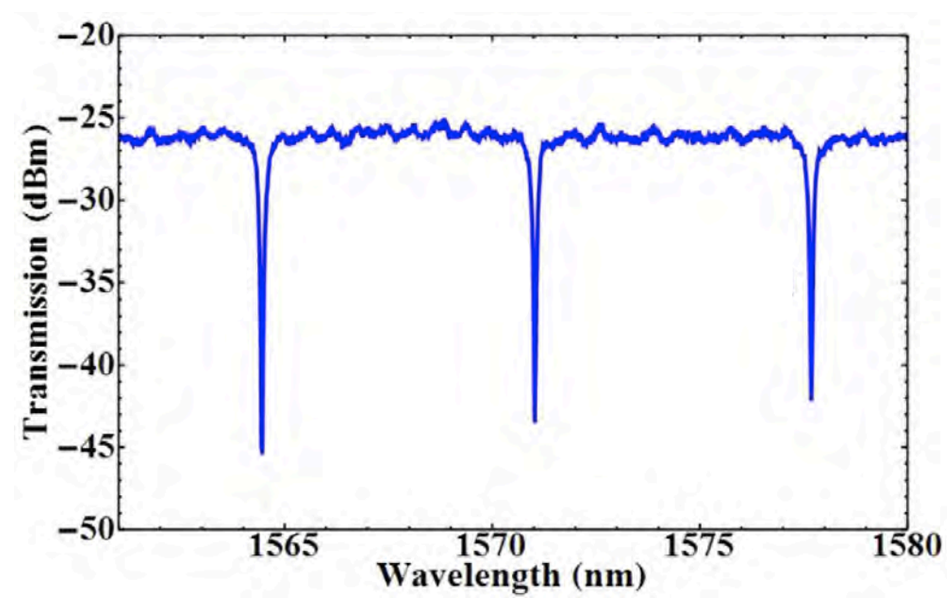
Why microring resonators?

- High efficiency due to small mode volume and high Q
- High FSR due to small size
- Small bandwidth
- CMOS-compatible manufacturing

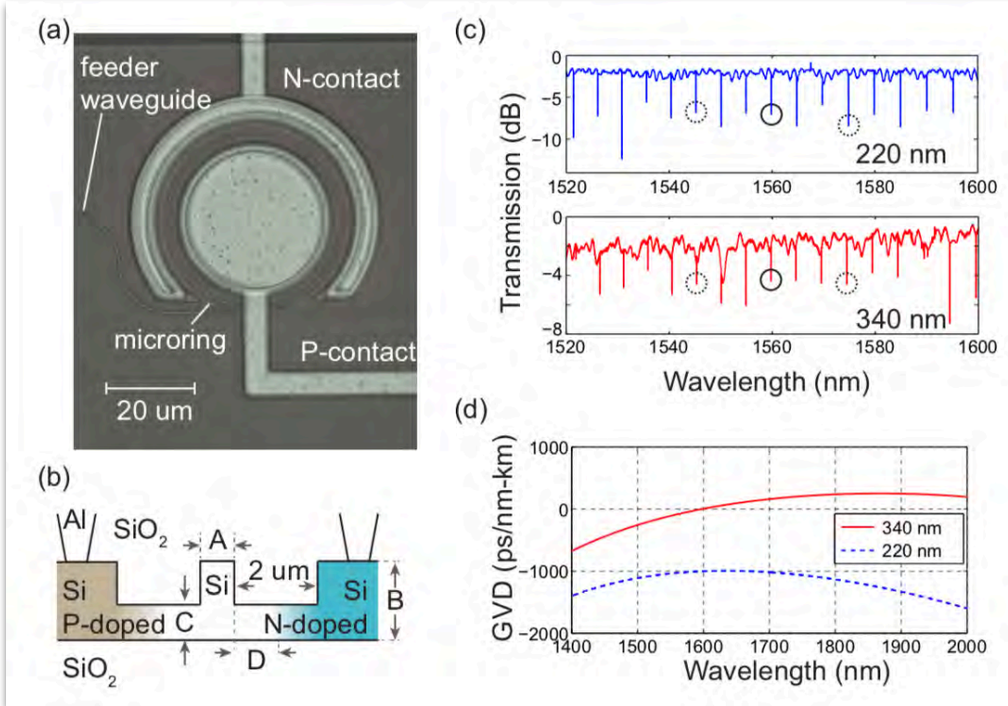




$$N \sim \gamma^2 \sim 1/A_{\text{eff}}^2$$

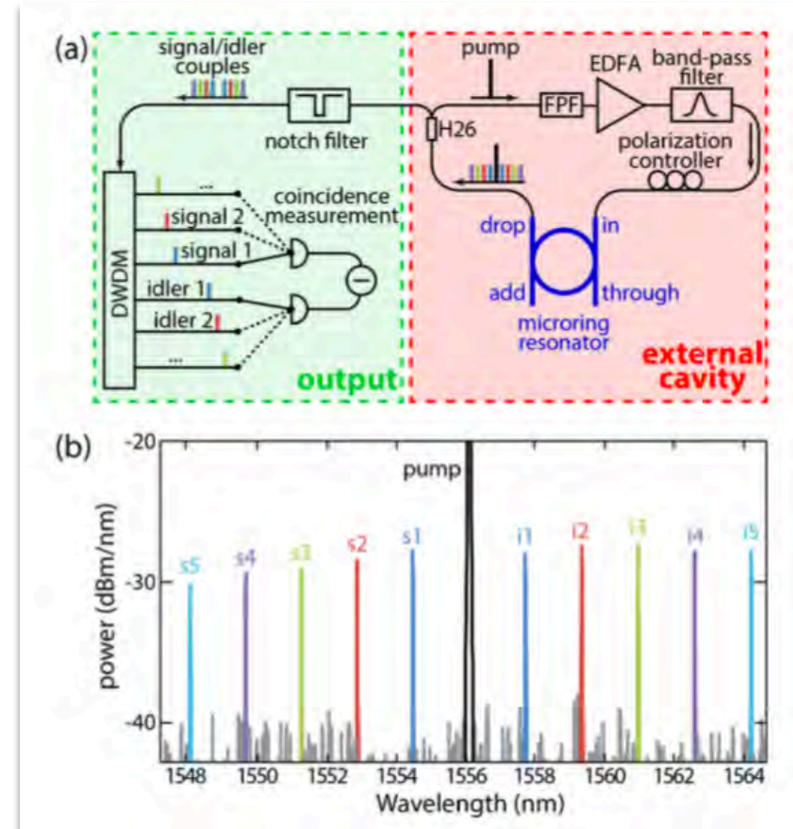


$$N \sim \sqrt{F_p F_p F_s F_i}$$



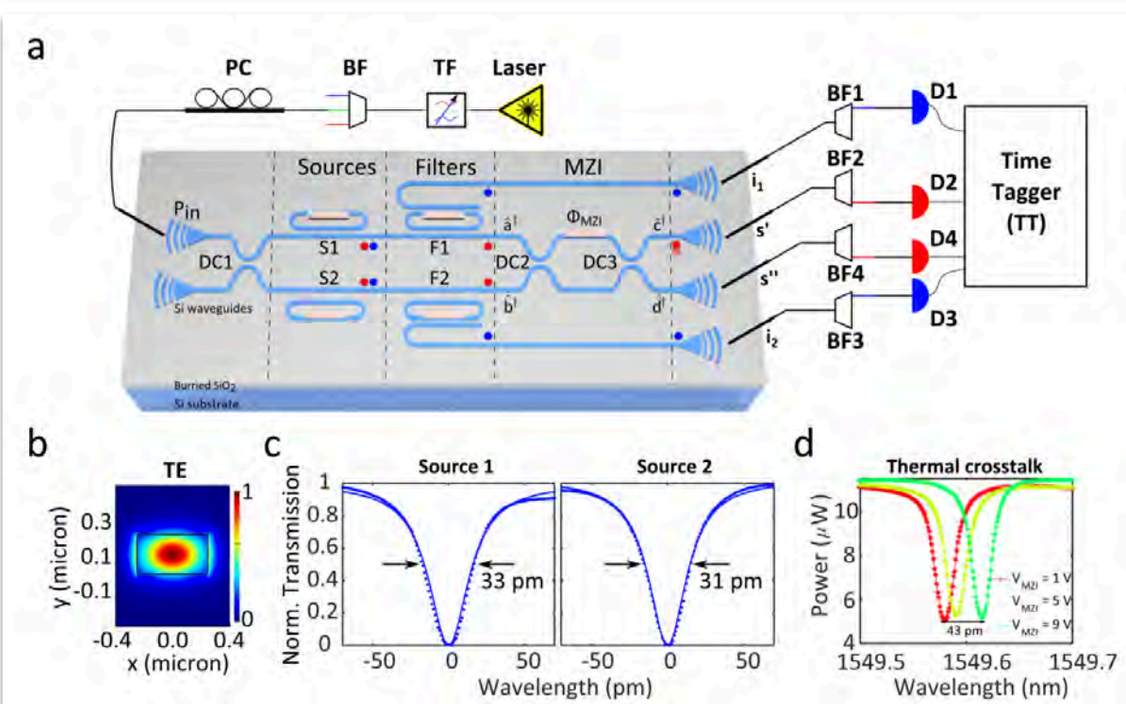
~ 100 kHz at ~10 μW pump

Savanier M, et al. Opt. Express 24 3313 (2016)



110 MHz bandwidth

Reimer C, et al. Opt. Express 22 6535 (2014)

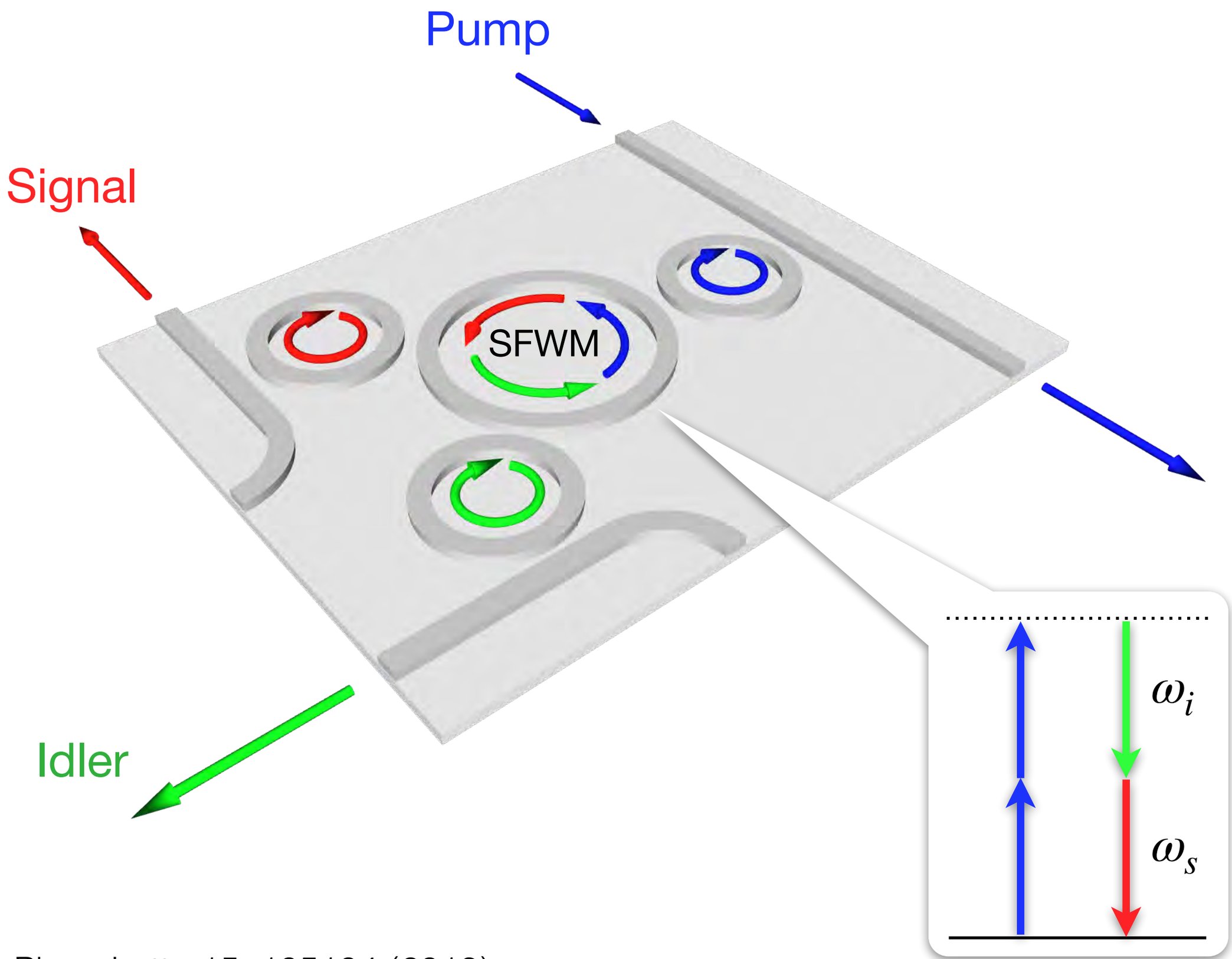


$$P = Sp(\rho^2)$$

Heralded photons
from independent sources
with the purity of 0.92

Faruque I.I., et al. Opt. Express 26 20379 (2018)

The model



Laser Phys. Lett., 15, 105104 (2018)

The model

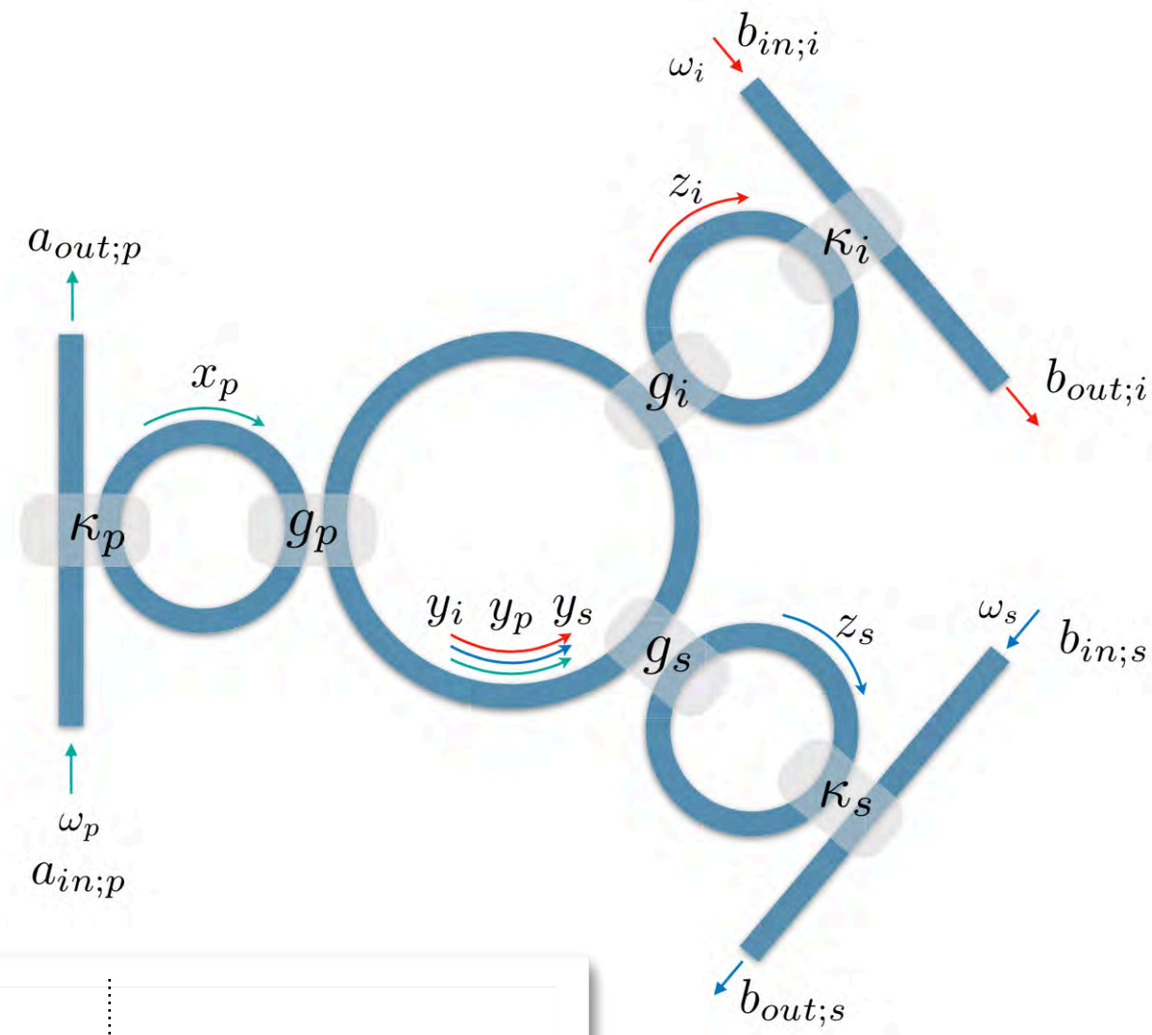
$$\mathcal{H}_{sys} = \hbar\omega_{0x,p}\, x_p^\dagger x_p + \sum_{n=p,i,s} \hbar\omega_{0y,n}\, y_n^\dagger y_n + \sum_{m=i,s} \hbar\omega_{0z,m}\, z_m^\dagger z_m$$

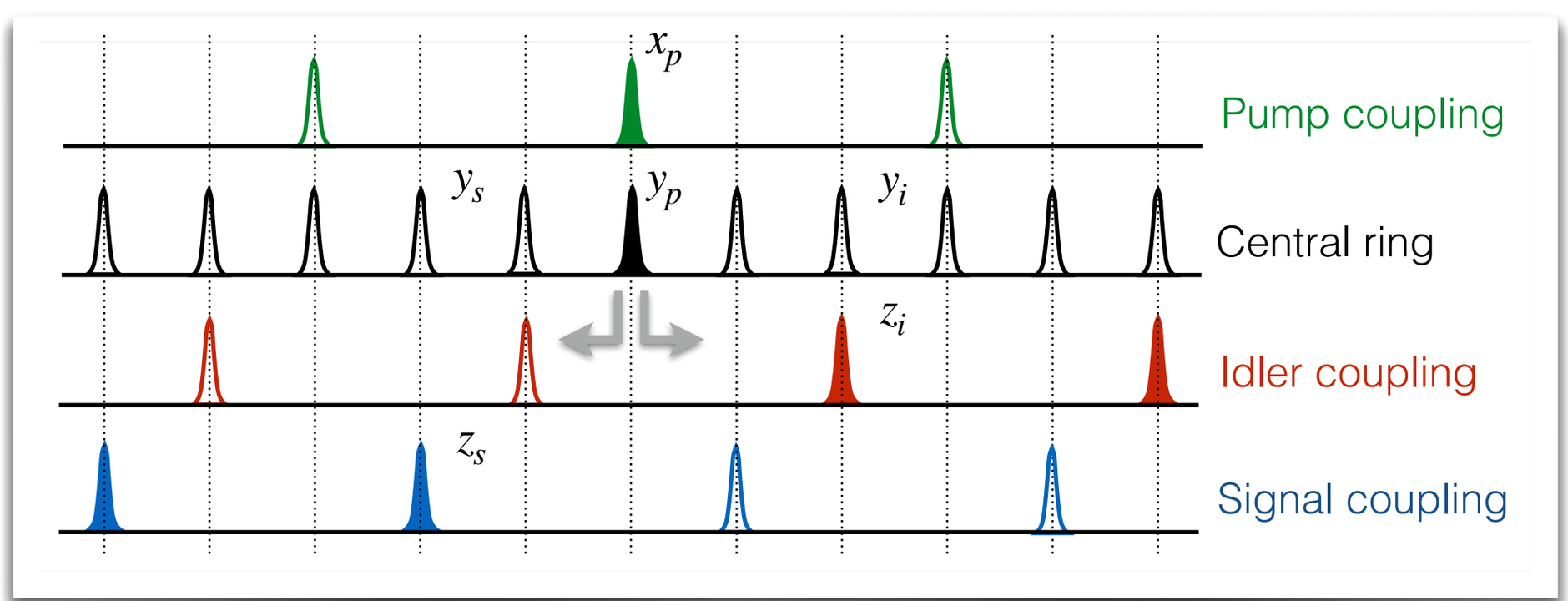
$$\mathcal{H}_{bath} = \int d\omega\, \hbar\omega \left[a_p^\dagger(\omega) a_p(\omega) + \sum_{m=i,s} b_m^\dagger(\omega) b_m(\omega) \right]$$

$$\mathcal{H}_{int}^{internal} = i\hbar g_p x_p^\dagger y_p + i\hbar g_i z_i^\dagger y_i + i\hbar g_s z_s^\dagger y_s + h.c.$$

$$\mathcal{H}_{int}^{bath} = \frac{i\hbar}{\sqrt{2\pi}} \int d\omega \left[\sqrt{\kappa_p}\, x_p^\dagger a_p(\omega) + \sqrt{\kappa_i}\, z_i^\dagger b_i(\omega) + \sqrt{\kappa_s}\, z_s^\dagger b_s(\omega) + h.c. \right]$$

$$\mathcal{H}_{SFWM}(t) = \zeta y_p(t) y_p(t) y_s^\dagger(t) y_i^\dagger(t),$$



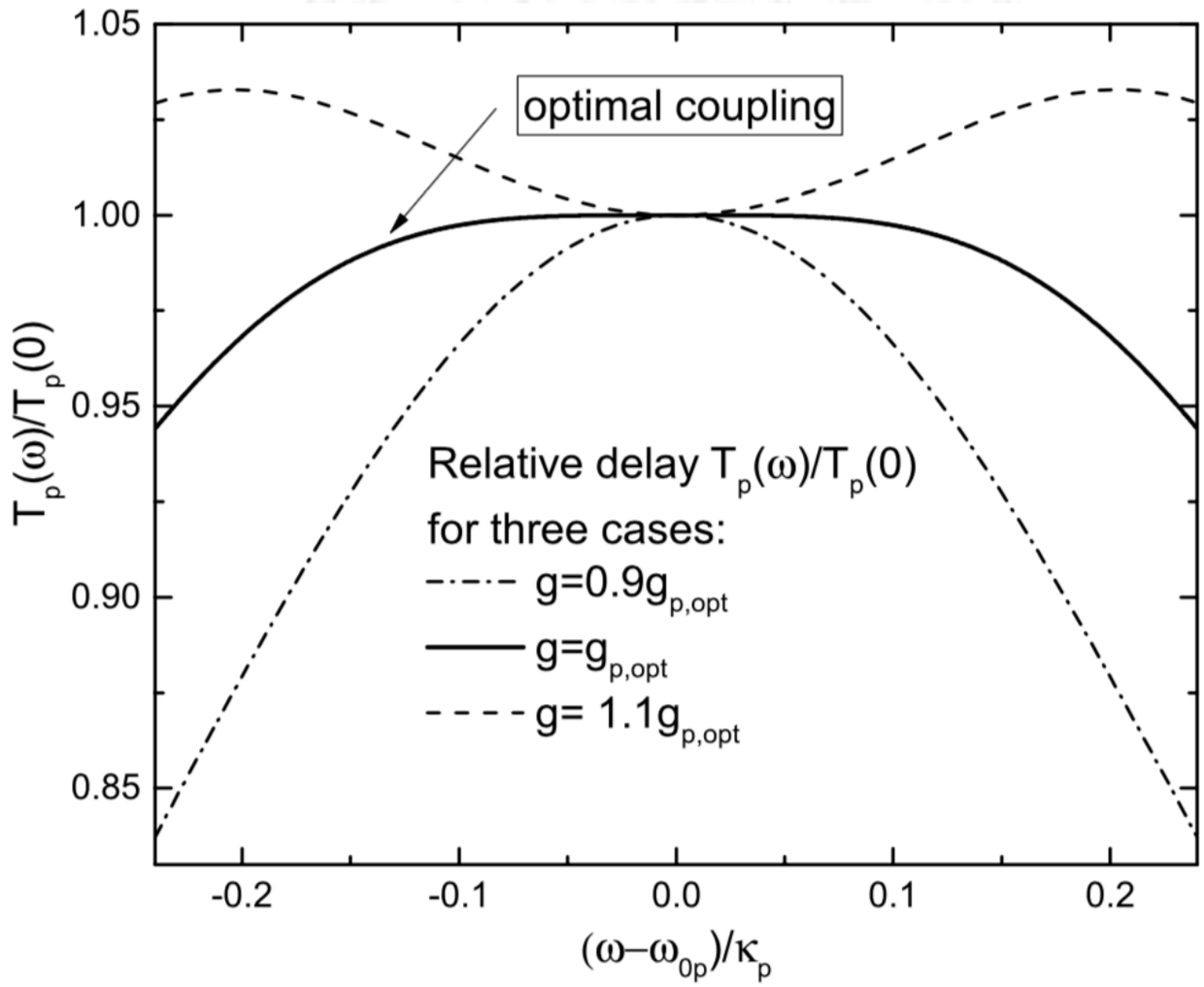


Optimal coupling

$$\partial_\omega^l \mathrm{Argument}(M_n)\Big|_{\omega=\omega_{0n}} = 0, \quad l = 2, 3, \ldots.$$

$$g_{p,\mathrm{opt}} = \kappa_p/\sqrt{12}, \; g_{is,\mathrm{opt}} = \kappa_{is}/\sqrt{12}.$$

$$T_p(\omega) = \mathrm{Argument}(M_p)/(\omega - \omega_{0p})$$



Outline of calculation

$$\mathcal{H}_{SFWM}(t) = \zeta y_p(t) y_p(t) y_s^\dagger(t) y_i^\dagger(t)$$

$$u(t) = \frac{1}{\sqrt{2\pi}} \int d\omega \, e^{-i\omega t} u(\omega)$$

$$y_p(\omega) = M_p \, a_{in;p}(\omega)$$

$$y_i(\omega) = M_i^* \, a_{out;i}(\omega) \qquad y_s(\omega) = M_s^* \, a_{out;s}(\omega)$$

$$|\psi\rangle = [1 - i/\hbar \int dt \, \mathcal{H}_{SFWM}(t)]|0\rangle|\alpha\rangle$$

$$|\psi\rangle = |0\rangle|\alpha\rangle - \frac{i\zeta}{\hbar\sqrt{2\pi}^3} \int d\omega_i d\omega_s \underline{\mathcal{F}(\omega_i, \omega_s)} \, y_{out;i}^\dagger(\omega_i) y_{out;s}^\dagger(\omega_s)|0\rangle|\alpha\rangle$$

Purity of the state

Joint Spectral Amplitude:

$$\mathcal{F}(\omega_i, \omega_s) = \mathcal{I}_p(\omega_i, \omega_s) M_i(\omega_i) M_s(\omega_s)$$

$$\mathcal{I}_p(\omega_i, \omega_s) = \int d\omega_p M_p(\omega_s + \omega_i - \omega_p) M_p(\omega_p)\, \alpha(\omega_s + \omega_i - \omega_p) \alpha(\omega_p)$$

Schmidt decomposition:

$$\mathcal{F}(\omega_i, \omega_s) = \sum_n \sqrt{\lambda_n}\, \psi_n(\omega_i) \phi_n(\omega_s), \quad \sum_n \lambda_n = 1$$

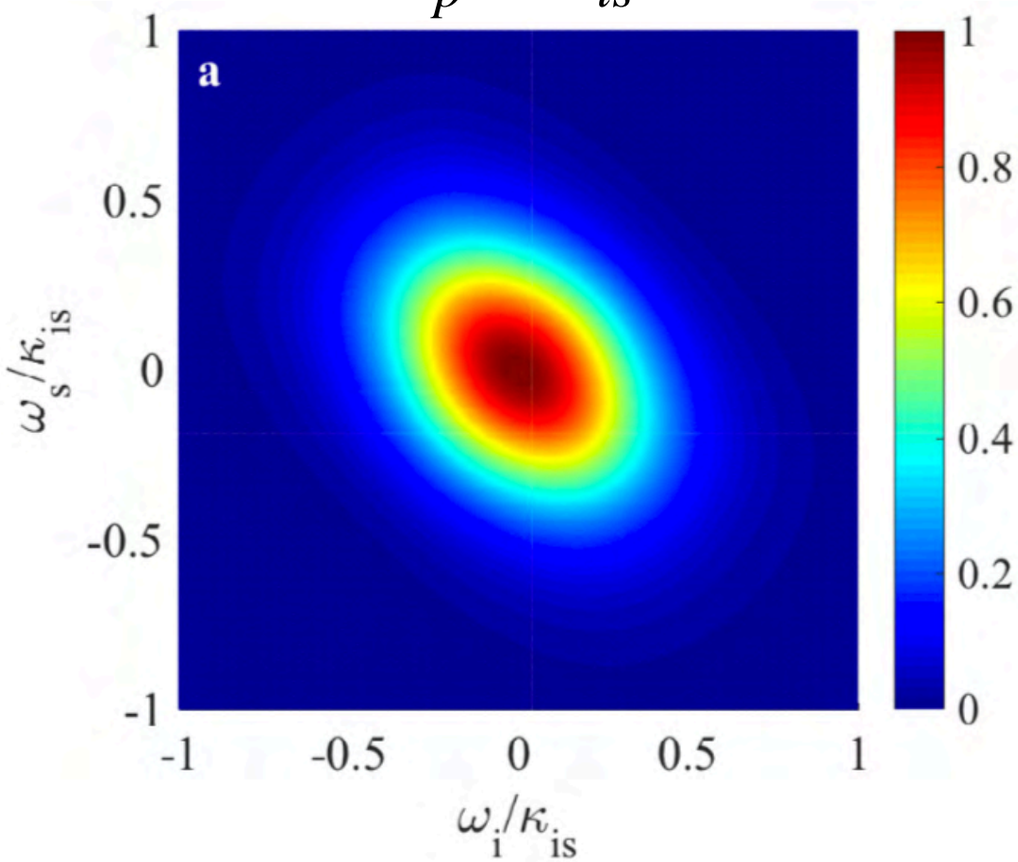
Schmidt number: $K = 1/\sum_n \lambda_n^2 \quad (K \geq 1)$

Purity: $P = 1/K \quad (0 \leq P \leq 1)$

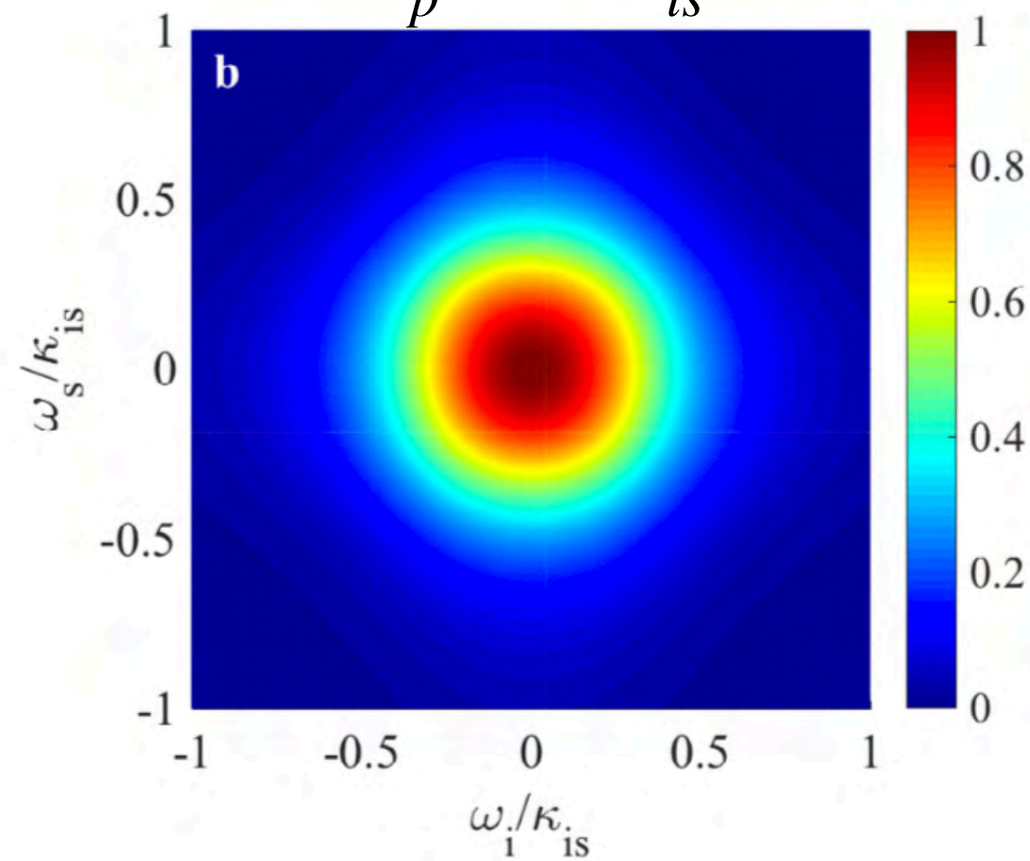
$K = 1 \;\rightarrow\;$ Factorable JSA $\;\rightarrow\;$ Pure state of heralded photons

Purity of the state

$\kappa_p = \kappa_{is}$



$\kappa_p = 10\,\kappa_{is}$



$K = 1.07$

$P = 0.94$

$K = 1.00006$

$P = 0.9999$

- Optimal ratios between coupling constants
- Gaussian pump pulses with optimal spectral width $\Delta\omega_{1/2} \approx \kappa_p/2$

Laser Phys. Lett., 15, 105104 (2018)

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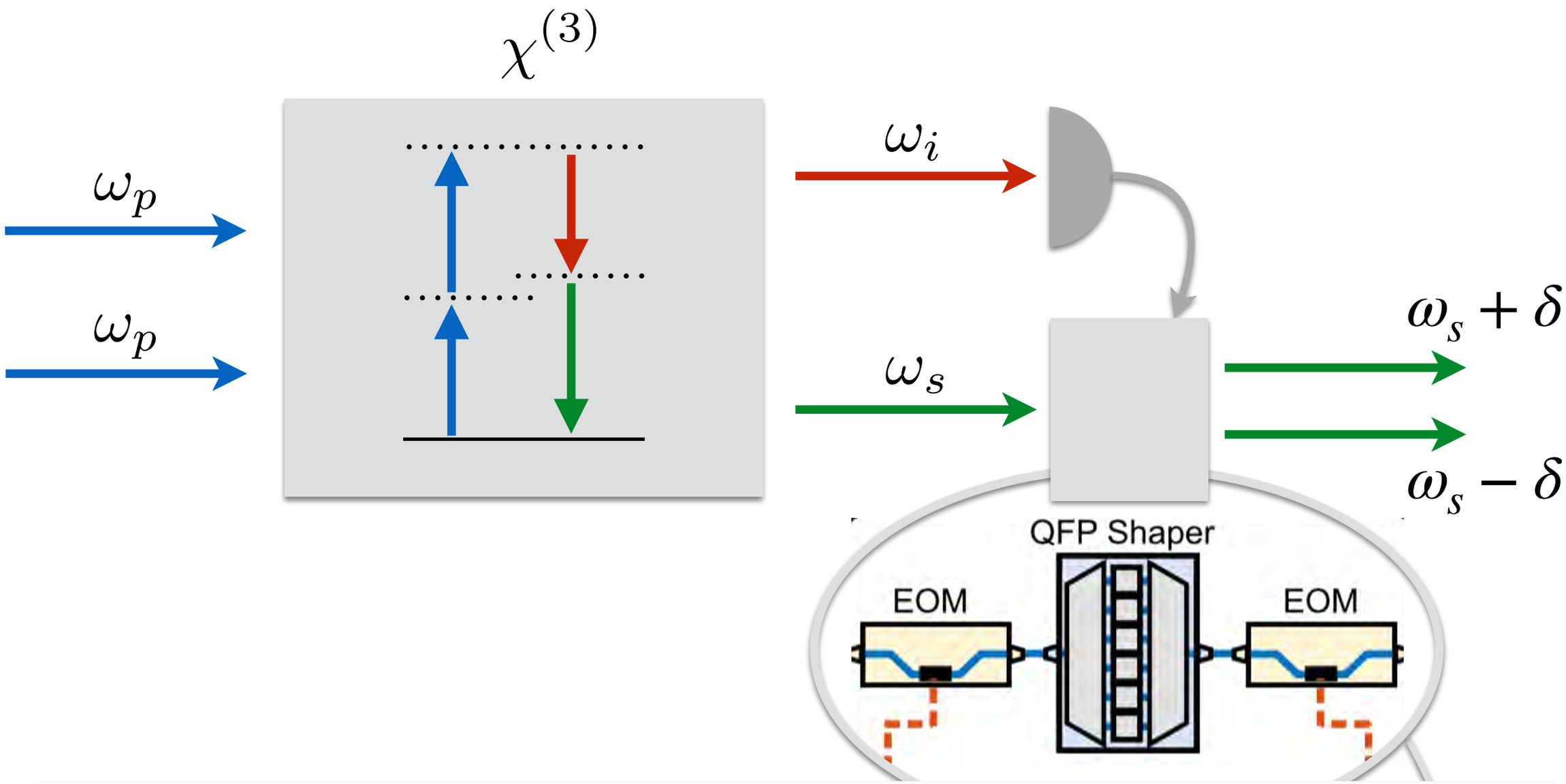
  Generating frequency-bin qubits
- Conclusion

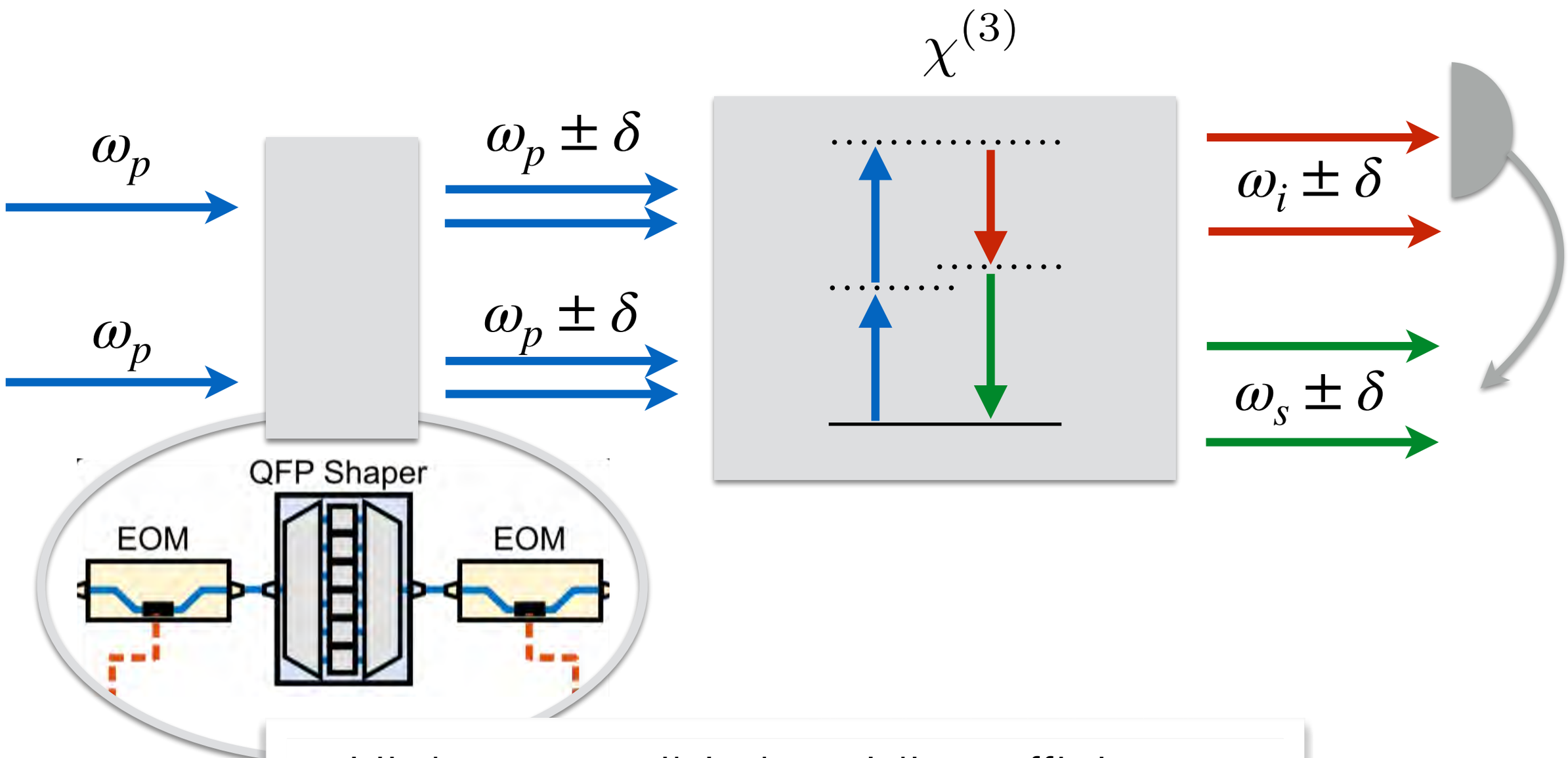
The simplest approach: modulation at the output



The heralding efficiency is reduced by insertion losses and by less than unit success probability of the gate

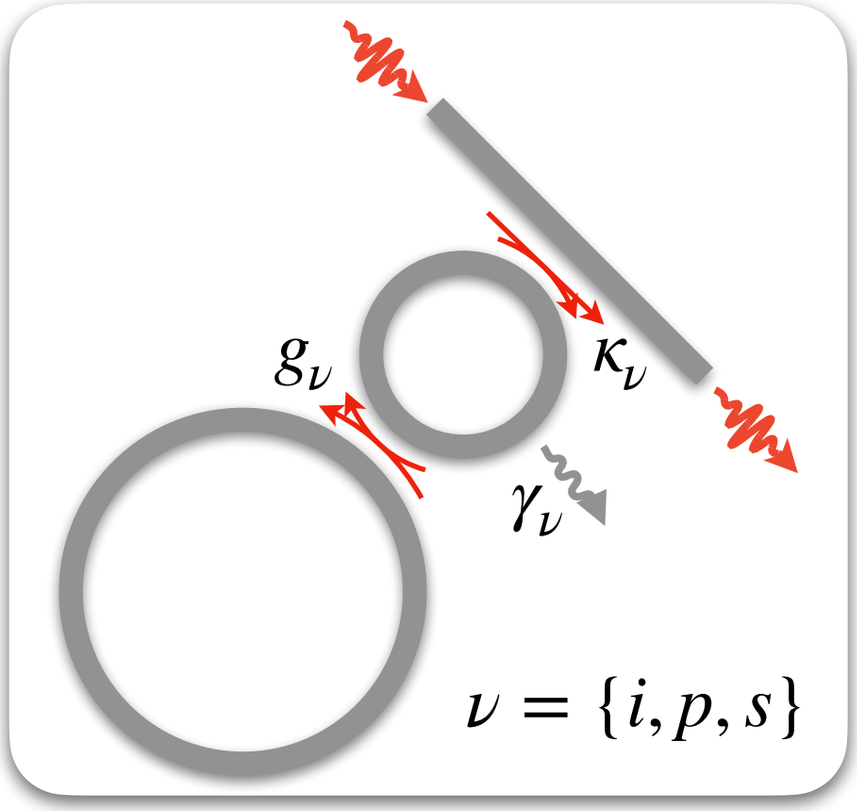
Single-qubit gate in the frequency domain:
PRL 120, 030502 (2018); PRL 125, 120503 (2020)

Our approach: modulation of the pump field



Highest possible heralding efficiency

Phys. Rev. A, 100, 043843 (2019)

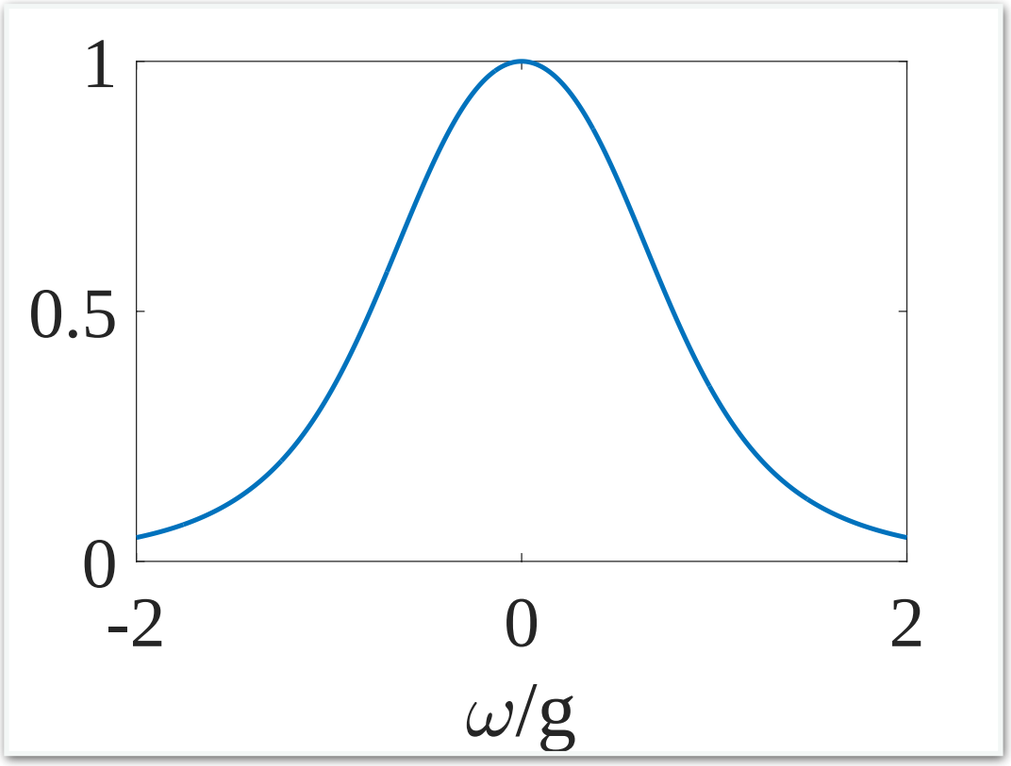


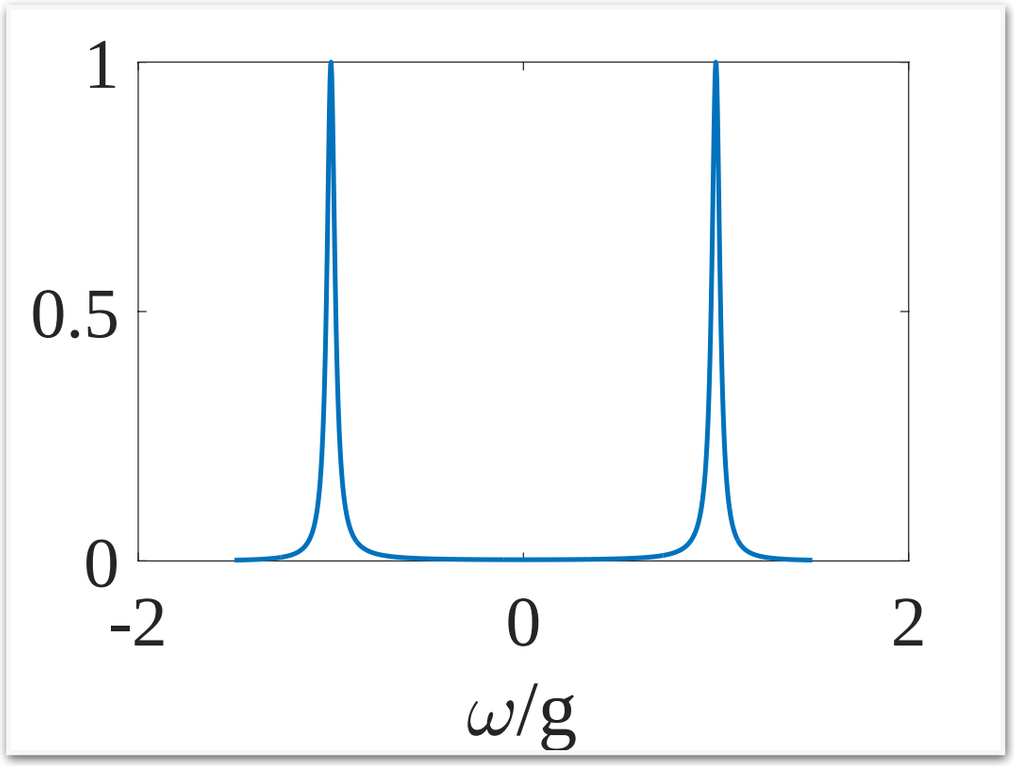
Weak coupling

$g_\nu \ll \kappa_\nu$

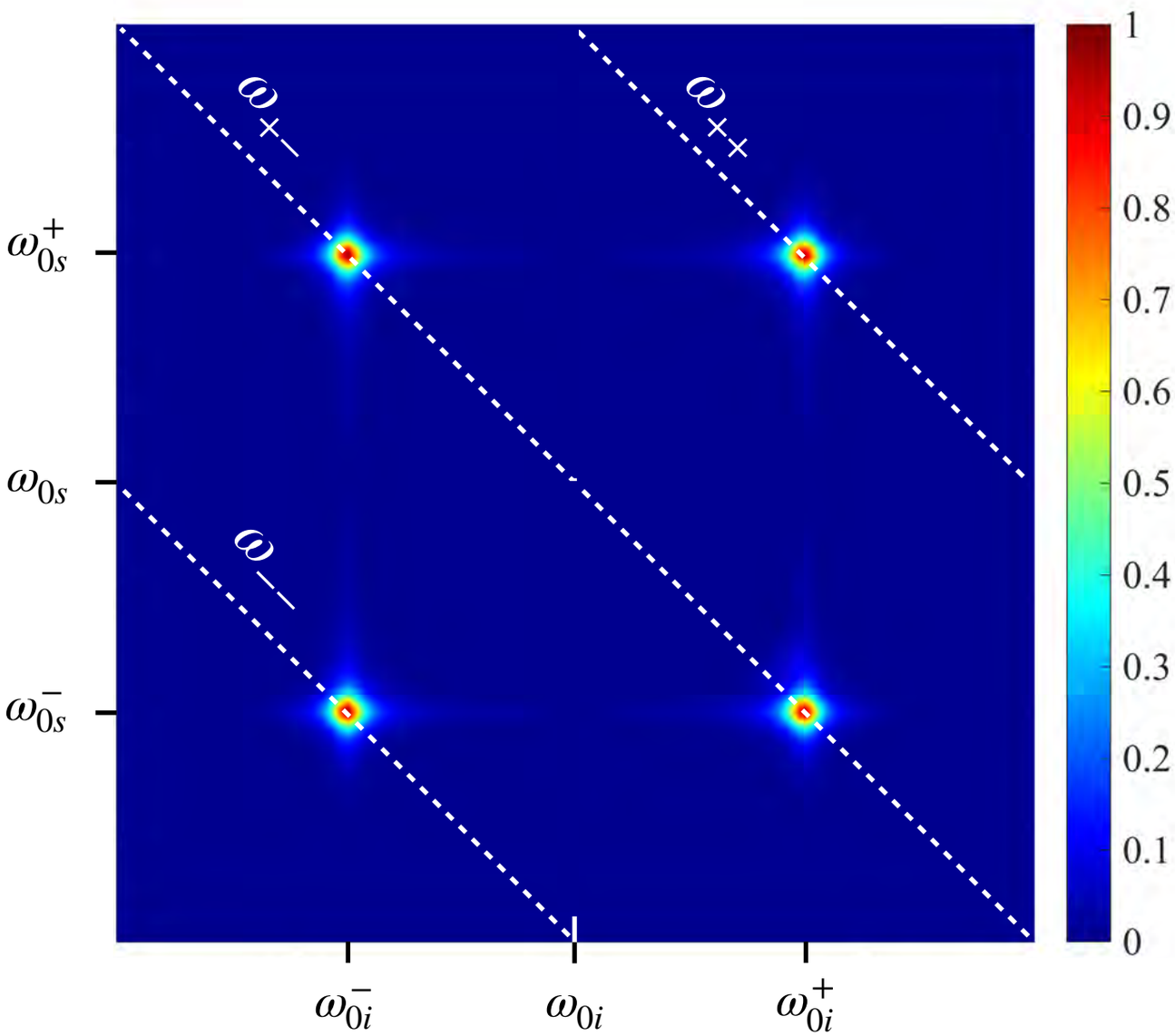
Strong coupling

$g_\nu \gg \kappa_\nu$



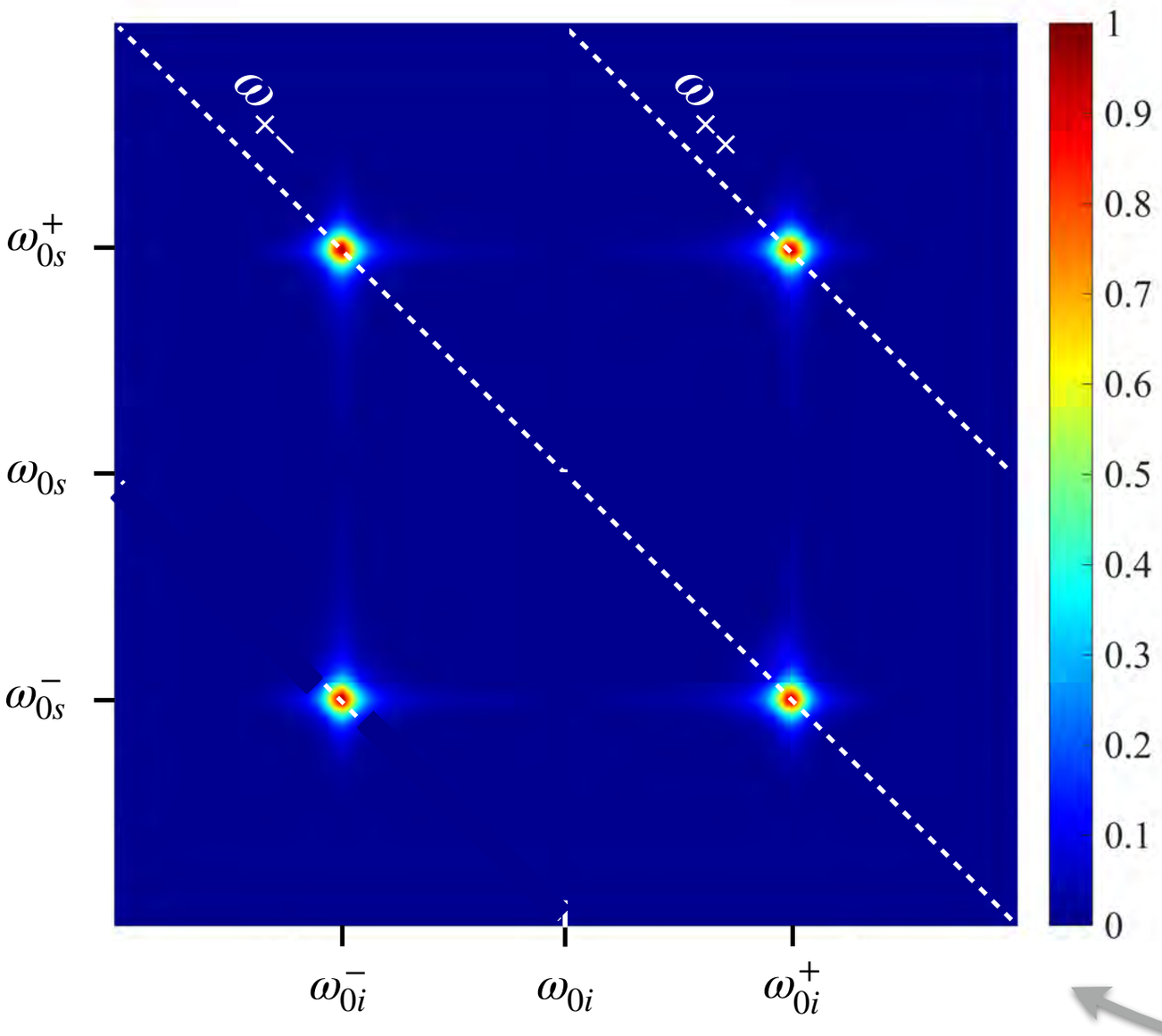


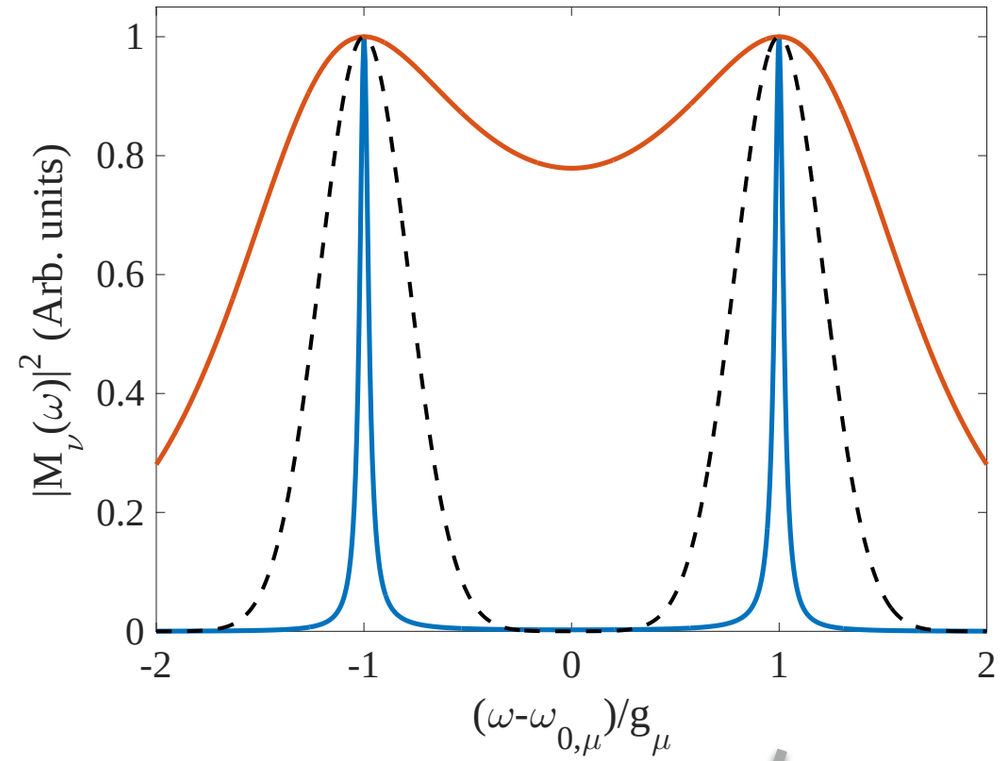
JSA: broadband pump field



$$M_\mu(\omega_\mu) = M_\mu^-(\omega_\mu) + M_\mu^+(\omega_\mu),$$

JSA: two-frequency pump field

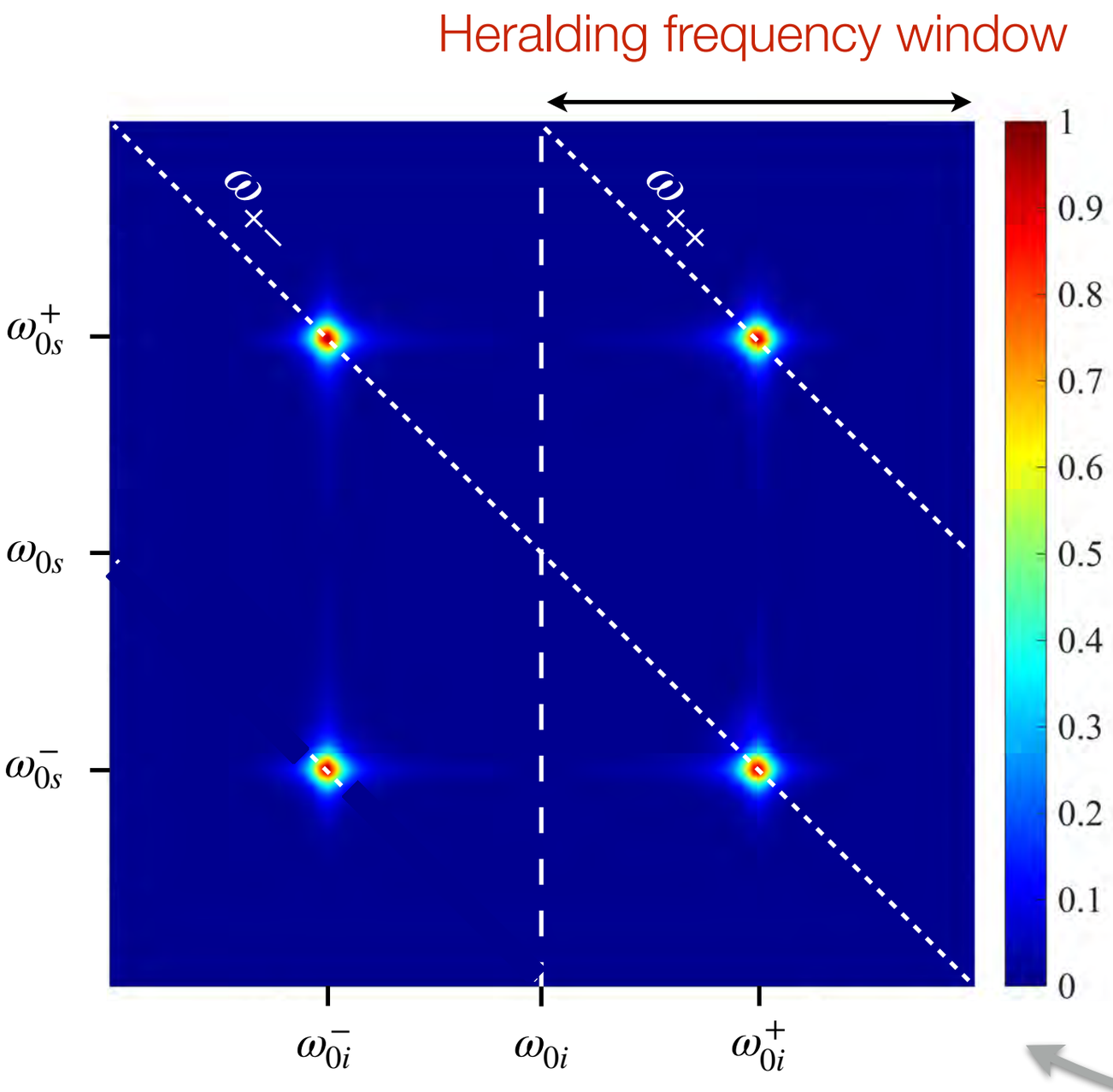


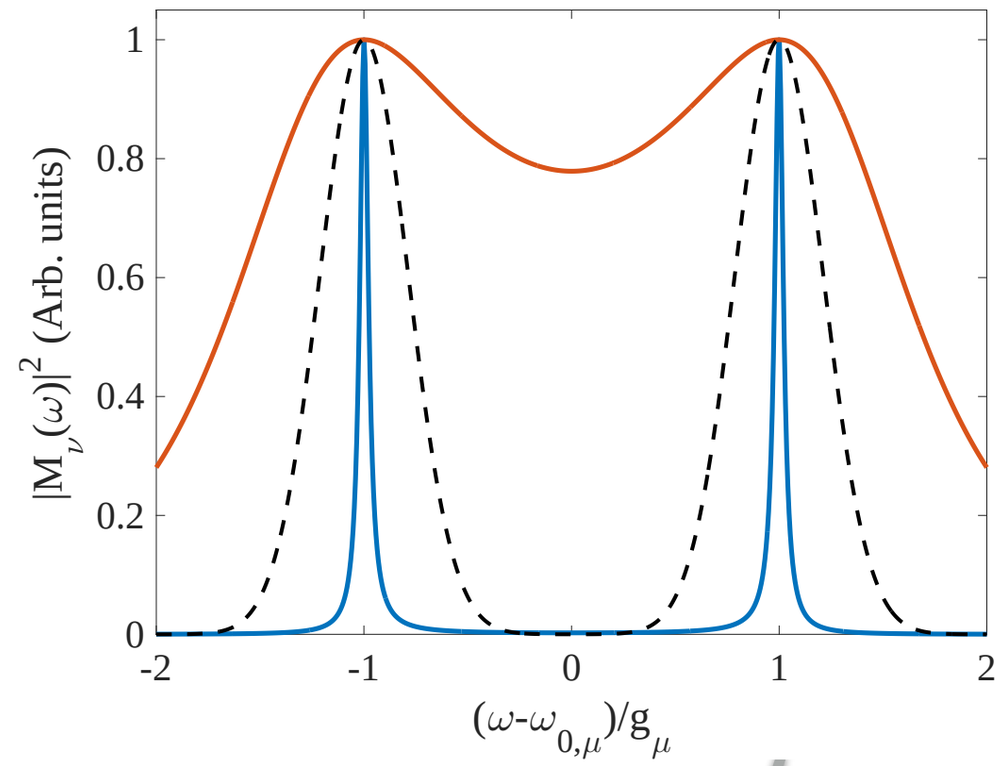


$$M_\mu(\omega_\mu) = M_\mu^-(\omega_\mu) + M_\mu^+(\omega_\mu),$$

$$\alpha(\omega_p) \sim \sqrt{A}\, e^{-\sigma(\omega_p-\omega_{+-})^2 - i\phi_a/2} + \sqrt{B}\, e^{-\sigma(\omega_p-\omega_{++})^2 - i\phi_b/2}$$

JSA: two-frequency pump field





$$|\psi_s\rangle = Ae^{-i\phi_a}|a\rangle + Be^{-i\phi_b}|b\rangle$$

$$|a\rangle = \int d\omega_s M_s^-(\omega_s) y_{out,s}^\dagger(\omega_s)|0\rangle$$

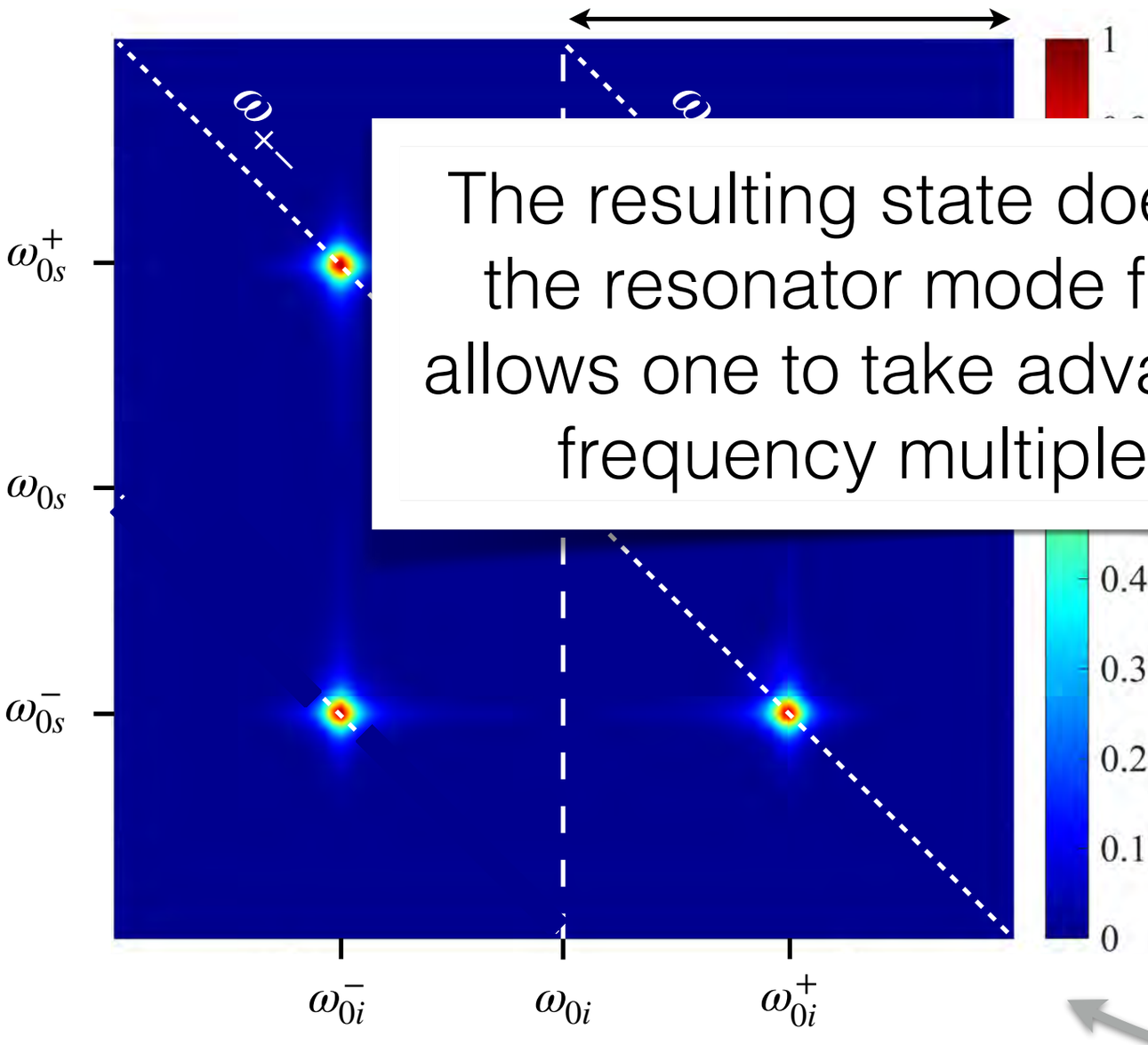
$$|b\rangle = \int d\omega_s M_s^+(\omega_s) y_{out,s}^\dagger(\omega_s)|0\rangle$$

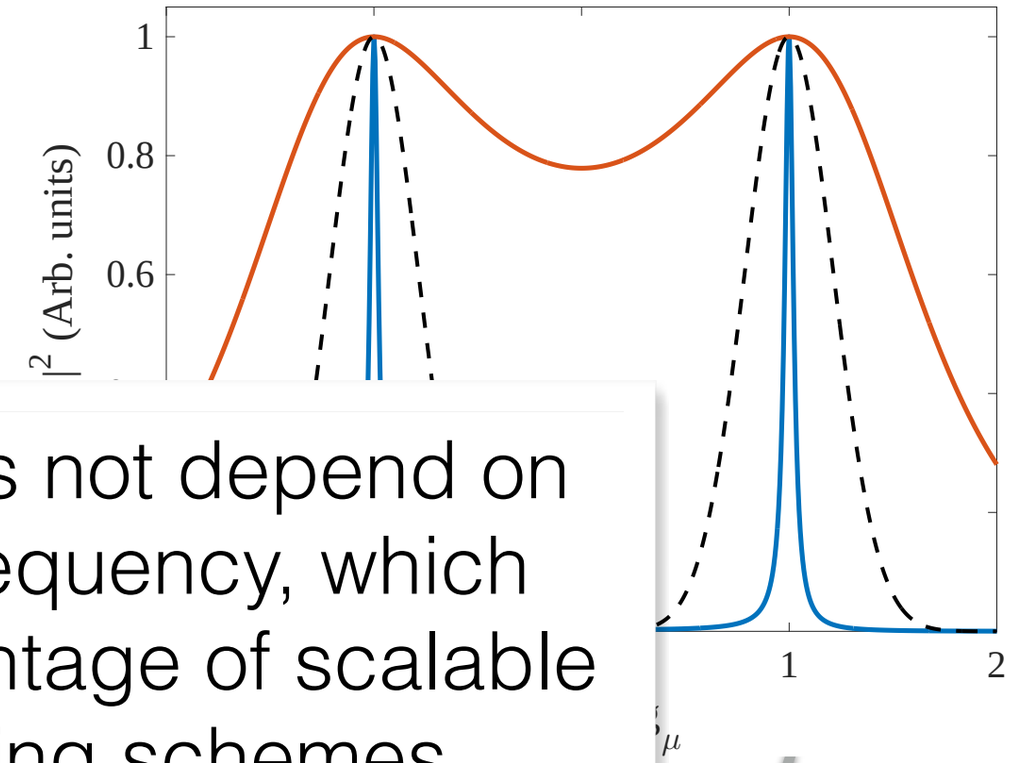
$$M_\mu(\omega_\mu) = M_\mu^-(\omega_\mu) + M_\mu^+(\omega_\mu),$$

$$\alpha(\omega_p) \sim \sqrt{A}\, e^{-\sigma(\omega_p-\omega_{+-})^2 - i\phi_a/2} + \sqrt{B}\, e^{-\sigma(\omega_p-\omega_{++})^2 - i\phi_b/2}$$

JSA: two-frequency pump field

Heralding frequency window





The resulting state does not depend on the resonator mode frequency, which allows one to take advantage of scalable frequency multiplexing schemes

$$|\psi_s\rangle = Ae^{-i\phi_a}|a\rangle + Be^{-i\phi_b}|b\rangle$$

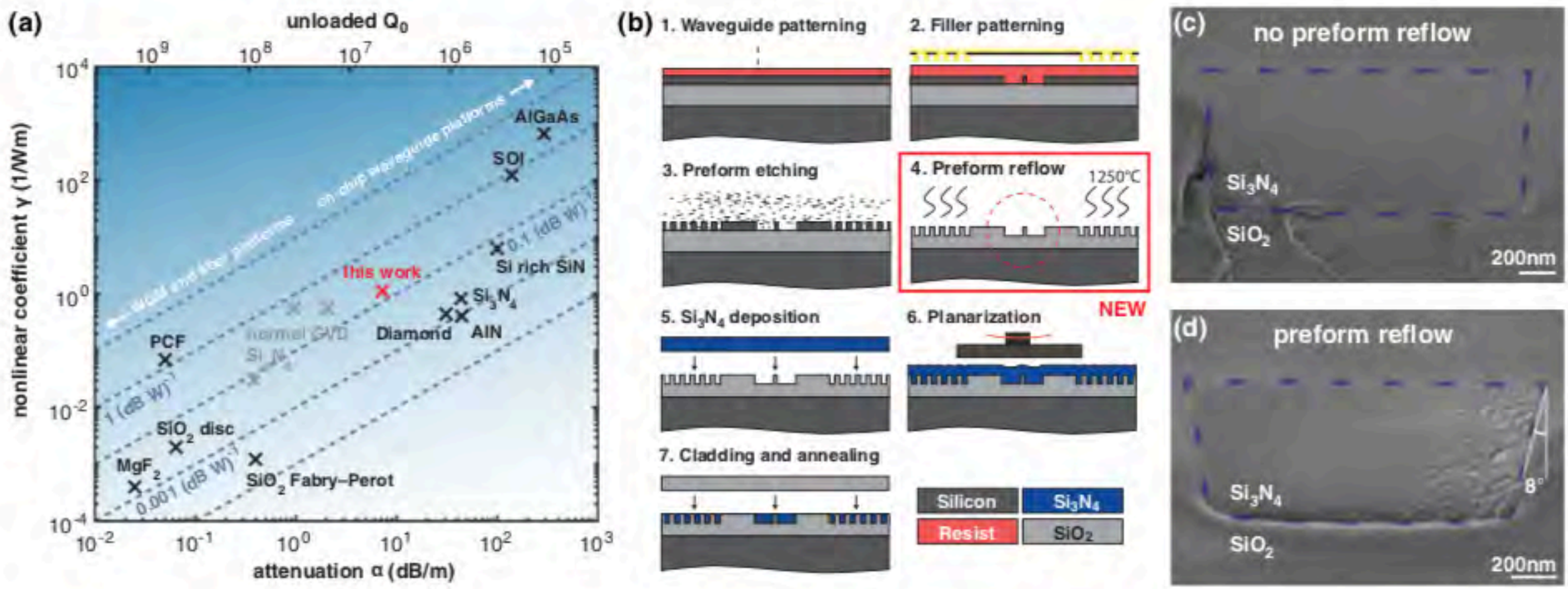
$$|a\rangle = \int d\omega_s M_s^-(\omega_s) y_{out,s}^\dagger(\omega_s)|0\rangle$$

$$|b\rangle = \int d\omega_s M_s^+(\omega_s) y_{out,s}^\dagger(\omega_s)|0\rangle$$

$$M_\mu(\omega_\mu) = M_\mu^-(\omega_\mu) + M_\mu^+(\omega_\mu),$$

$$\alpha(\omega_p) \sim \sqrt{A}\, e^{-\sigma(\omega_p-\omega_{+-})^2 - i\phi_a/2} + \sqrt{B}\, e^{-\sigma(\omega_p-\omega_{++})^2 - i\phi_b/2}$$

Silicon Nitride (Si_3N_4) ring resonators

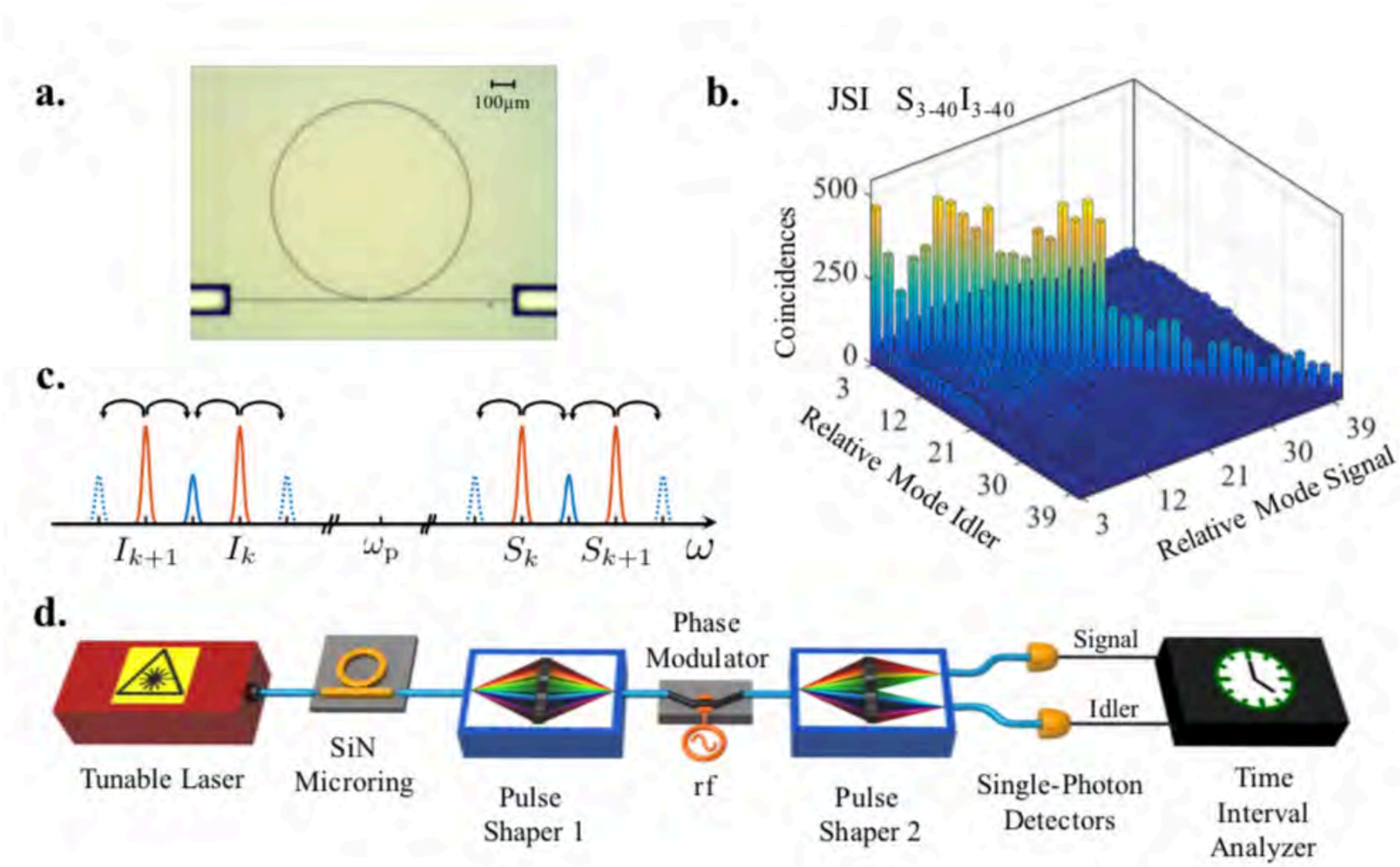


Losses ~ 0.01 dB/cm at 1.5 um

$Q > 1 \cdot 10^7$, $R = 230\ \mu\mathrm{m}$, $F \sim 3000$

M.H.P. Pfeiffer, et al, Optica 5, 884 (2018)

Silicon Nitride (Si_3N_4) ring resonators



~ 40 entangled modes with frequency interval ~ 50 GHz
Q ~ 10^6, $R = 500\ \mu\mathrm{m}$

P. Imany, et al. Opt. Express 26, 1825 (2018)

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- Single-photon qubits can be prepared via optical nonlinear effects with highest possible heralded efficiency by manipulating the pump field alone.

- The corresponding schemes have been developed for time-bin qubits generated via SPDC in a cavity [1] and frequency-bin qubits generated via SFWM in a system of microresonators [2]

- In the spatial domain, quantum hashing based on the preparation of single-photon qubits in the basis of orbital angular momentum by manipulating the pump field has been implemented [3]. Cavity-based variant of spatial qubit preparation is still to be done.

[1] Phys. Rev. A, 81, 043809 (2010)
[2] Laser Phys. Lett., 15, 105104 (2018)
[3] Phys. Rev. A, 100, 043843 (2019)
[4] Phys. Rev. A, 104, 052606 (2021)

Acknowledgments

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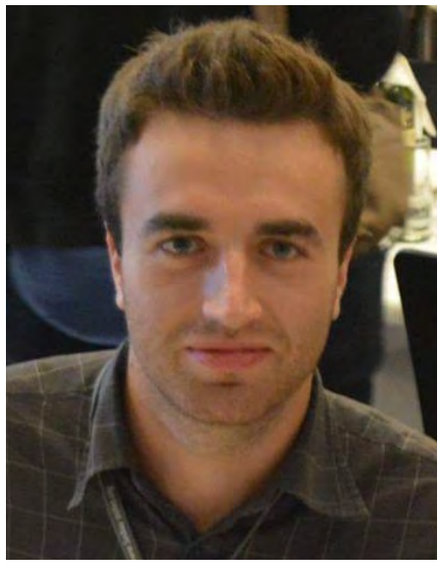

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