Two-Page Summary Preparation for QTS. Important: Do Not Use Symbols, Special Characters, or Math in the title

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The work is devoted to the development of framework for probability representation of finite-dimensional quantum mechanics using informationally complete positive operator-valued measures(IC-POVMs). In this representation quantum states are fully described by vector distribution, and system's dynamics is described by the action of pseudostochastical matrices on the vector-distributions. Earlier, such a representation was studied in case of symmetrical IC-POVMs (SIC-POVMs) [?, ?, ?, ?]. The generalisation on non-symmetric case is presented.

Consider POVM $E = (E_k)_k$, a set of positive operators E_k, which sum up to identity: $\sum_k E_k = I$. The notion of informationally completeness implies that E is a linear basis, so each quantum state ρ may be represented as a sum

$$\rho = \sum_k p_k \kappa_k, \quad p_k = \mathrm{tr}(\rho E_k),$$

where $\kappa = (\kappa_l)_l$ is a linear basis of operators, such that $\mathrm{tr}(\kappa_l E_k) = \delta_{lk}$, and $p = (p_k)_k$ is a vector-distribution. This way, it is possible to embed a space of all quantum states in the simplex of distributions. Note that not each state is described by a distribution. The subset of distributions, which do correspond to states, was studied in SIC-POVM case and was called *qplex* [?].

To each quantum channel Φ (completely positive trace preserving map) corresponds a pseudostochastic matrix S with elements

$$S_{lk} = \mathrm{tr}(E'_l \Phi[\kappa_k]).$$

Pseudostochasticity of a matrix S means that it's columns sum up to 1, but it may have negative elements, unlike a stochastic matrix. Given a pseudostochastic map S the action of a corresponding channel Φ is given by a formula

$$\Phi[\rho] = \sum_{l,k} S_{lk} \kappa'_l \mathrm{tr}(E_k \rho).$$

In the work the algorithm for checking, whether a given distribution p corresponds to some state ρ, was proposed. The idea of the method is to find a characteristic polynomial of ρ and check if all roots are non-negative. Also, using this algorithm together with Choi-Jamilkowsky isomorphism, it is possible to check whether a given pseudostochastic matrix S corresponds to a quantum channel Φ.

The markovian dynamics of open quantum systems are given by Gorrini-Kossakowsky-Sudarshan-Lindblad (GKSL) equations

$$\frac{d}{dt}\rho = -i[H;\rho] + \Psi[\rho] + \frac{1}{2}\{\Psi^*[I];\rho\}.$$

Here Ψ is a completely positive linear map, not necessary trace preserving. The same equations in probability representation takes a form

$$\frac{d}{dt}p = (\mathbf{H} + \mathbf{D})p,$$

where $\mathbf{H}_{lk} = i\mathrm{tr}(E_l[H;\kappa_k])$ is an orthogonal matrix describing unitary evolution under hamiltonian H, and

$$\mathbf{D}_{lk} = S_{lk} + \sum_{n,m} \frac{\epsilon_k^{ln} + \epsilon_k^{nl}}{2} \quad (\epsilon_k^{ln} = \mathrm{tr}(E_l E_n \kappa_k)),$$

is a dissipator matrix.

The received results are of interest in the fields of modelling quantum systems via classical stochastic values and general development of quantum theory.

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