Entanglement between identical particles is a useful and consistent resource[1]

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Identical particles in quantum mechanics have a character quite distinct from those in classical mechanics. Classically, indistinguishability comes from limited abilities of the experimenter; in the quantum world, two particles of the same type, such as electrons, are fundamentally indistinguishable [2, 3]. This feature applies not only to fundamental particles but is also crucial in describing identical composite particle systems such as Bose-Einstein condensates (BECs) [4]. Notably, exchanging two identical quantum particles results in an overall phase change in the wavefunction: no change for bosons and a minus sign for fermions.

These exchange statistics require a symmetric or anti-symmetric wavefunction in the first-quantised formalism. For example, let us denote by $|n_0, n_1\rangle$ a state of identical bosons in which n_0, n_1 particles have the internal state $|0\rangle$, $|1\rangle$ respectively. In the first-quantised picture, we represent $|1,1\rangle$ not as a two-mode state but a symmetric two-particle state

$$\frac{|0\rangle_1|1\rangle_2 + |1\rangle_1|0\rangle_2}{\sqrt{2}}, \tag{1}$$

in which we have attached the fictional labels $1, 2$ to the particles. Formally, the state (1) is entangled. However, it may be argued [5, 6, 7, 8, 9, 10] that this entanglement is unphysical — since the particles are identical, the labels $1, 2$ are meaningless as it is impossible to say which particle has which label. Throughout this work we will refer to this entanglement as *Particle Entanglement* (PE) [2].

A consensus on the nature of this entanglement has so far been out of reach [12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27]. Some authors view PE as a failure of the mathematical formalism and argue that it should be disregarded in favour of other definitions of identical-particle entanglement [13, 5, 6, 14, 15, 16, 17, 21, 9, 25]. One class of approaches requires talking only about correlations between observables [15, 16, 17, 21, 23, 25]; other authors pursue entirely new definitions of entanglement tailored to the identical-particle setting [12, 13, 6, 5, 14, 7].

In order to determine whether there is any meaningful interpretation of PE *per se* we follow the modern approach to entanglement within quantum information theory [28]. Here, entangled states are defined as those which cannot be prepared by two or more separated parties who are unable to send quantum information, and are as such limited to local operations (within their own laboratories) and classical communication — abbreviated as LOCC. Entanglement is then regarded as a *resource* for parties operating under such constraints, and can enable them to perform better at a vast range of tasks including quantum communication [29], computation [30], key distribution [31], and metrology [32], to name a few.

In order to justify PE as a resource, one needs to provide the appropriate setting — what is the analogue of LOCC for indistinguishable particles? In this work, we first answer that question by finding a physically relevant set of quantum operations in which PE cannot be created. These operations are constructed from combinations of appending vacuum states, performing passive linear unitaries and making either non-demolition measurements of total particle number, or else arbitrary but destructive measurements. We prove that each of these sets of elements is as general as possible while resulting in a consistent theory. In particular, the set of unitaries is physically motivated as "easy" in many settings, corresponding to beam splitters and phase shifters in optics, and to number-conserving non-interacting hamiltonians in condensed matter systems. These operations, which we call *particle-separable* $\mathcal{O}$, define the basis of a *resource theory* for PE. Such an approach has been widely employed recently to pin down a variety of quantum properties beyond entanglement, such as quantum thermodynamics [47], quantum

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[1] Full Text available here, [1]

[2] Not to be confused with particle entanglement as named in [11].

coherence [48] and asymmetry [49]. With this structure in place, one can begin to rigorously quantify PE.

We use this framework to find the complete setting in which PE is a resource for generating useful mode entanglement between parties. Specifically, by "useful" mode entanglement we mean that which is accessible to parties who are constrained not only by LOCC but also by a local particle-number superselection rule [51]. The latter constraint renders superpositions of different particle numbers unobservable, and applies when particle number is conserved and the two parties do not have access to a shared phase reference [52]. Under this limitation, less entanglement can be utilised [34, 11]. We show that useful entanglement can be generated from an initial state by a particle-separable operation exactly when the initial state contains non-zero PE. Furthermore, we find quantitative relations between the amount of input PE and the output useful entanglement. This shows that PE mirrors other quantum resources which may be similarly "activated" into useful entanglement [53, 54, 55].

These results have direct applications to real systems of indistinguishable bosons, in particular entangled states of BECs [56, 27]. We analyse one of a set of recent experimental advances witnessing mode entanglement in BECs [57, 58, 59]. We show that these fit into our framework and implement the above resource conversion. In particular, our results enable a quantitative determination of the PE content of the states produced in the experiment.

In this work we have seen that entanglement between identical particles, despite its seemingly fictitious nature, is described by a consistent resource theory whose free operations are implementable in a wide range of physical systems. Far from just an abstract quantity, this entanglement can be activated, via the same types of operations, into directly accessible entanglement. This occurs in a setting where phase references are not easily shared between separated parties, enforcing a local SSR.

It is hoped that the results presented here and future theoretical and experimental studies utilising this framework will provide insight into the genuinely quantum properties and behaviour of such systems.

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