Finding the optimal cluster state configuration. Cluster states classification by type of computations

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One-way quantum computations (OWQC) with continuous-variables are a convenient alternative to computations performed with qubits [1]. All OWQC are realized through successive local measurements of a multipartite entangled state called a cluster state. However, despite the general principle of construction of computation, discrete and continuous systems are significantly different from each other. This distinction is based on the various properties of the quantum states of the physical systems that compose cluster nodes.

Physical systems in the quadrature-squeezed state (squeezed quantum oscillators) are used to generate continuous-variable clusters. Each such oscillator is defined by two quadratures $\hat{x}_s$ and $\hat{y}_s$, which obey the canonical commutation relation. The squeezing of oscillators means that the variance in one of the quadratures is smaller than the variance of the vacuum state. It is generally accepted to consider oscillators squeezed in $\hat{y}_s$-quadrature, $\langle \delta \hat{y}_s^2 \rangle < 1/4$.

In our work, we analyze all possible cluster configurations for the feasibility of universal Gaussian transformations on them. It is well-known [2, 3] that the result of such transformations can be written in vector form:

$$\begin{pmatrix} \hat{\vec{X}}_{out} \\ \hat{\vec{Y}}_{out} \end{pmatrix} = \tilde{U} \begin{pmatrix} \hat{\vec{x}}_{in} \\ \hat{\vec{y}}_{in} \end{pmatrix} + E\hat{\vec{y}}_s, \tag{1}$$

where $\tilde{U}$ is a main transformation matrix, E is an error matrix, $\hat{\vec{y}}_s$ — is a vector consisting of squeezed $\hat{y}$ - quadratures of all cluster nodes, $\hat{\vec{x}}_{in}$ and $\hat{\vec{y}}_{in}$ are input quadrature vectors over which the computation will be performed, and $\hat{\vec{X}}_{out}$, $\hat{\vec{Y}}_{out}$ are output quadrature vectors obtained as a result of computation. In Equation (1), the matrices E and $\tilde{U}$ depend on the cluster state configuration, because OWQC are implemented via local measurements of the entangled cluster state nodes. It was originally believed that the richer the cluster state configuration (i.e., the more nodes and edges in the cluster state), the more transformations could be performed with it. However, this approach is only correct when using quantum oscillators with infinite squeezing, which, of course, is not realistic. If we talk about physical systems, then an increase in the number of nodes leads to an increase in errors associated with the finiteness of oscillators squeezing [4]. As a result, this error can grow to the point that it will be impossible to correct, and it will ruin all computation. Thus, possessing the oscillators with some finite squeezing, we should optimize the configuration of the cluster state so that it remains suitable for universal Gaussian transformations and would provide the smallest computation errors.

In this work, we will solve two problems. First, we will select from all possible cluster configurations those that allow the implementation of universal Gaussian transformations. To that end, we will classify cluster states, and for each class, we will find general expressions of the type (1). Next, we will use the obtained equations to determine what transformations can be performed on these states and select the required configurations.

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References

[1] *R. Raussendorf, H. Briegel*, A One-Way Quantum Computer. Phys. Rev. Lett., **86**, P. 5188-5191, (2001).

[2] *R. Ukai, J. Yoshikawa, N. Iwata, P. van Loock, A. Furusawa*, Universal linear Bogoliubov transformations through one-way quantum computation. Phys. Rev. A, **81**, P. 032315, (2010).

[3] *S. Yokoyama, et al.*, Ultra-large-scale continuous-variable cluster states multiplexed in the time domain. Nat. Photon. **7**, P. 982, (2013).

[4] *S.B. Korolev, A.D. Manukhova, K.S. Tikhonov, T.Y. Golubeva, Y.M. Golubev*, Criteria of minimum squeezing for quantum cluster state generation. Laser Phys. Lett. **15**, P. 075203, (2018).